JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITYDRAFT EXAMINATION FOR BSc IN MATHEMATICS

1st YEAR 1st SEMESTER 2017/2018 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SMA 3113

COURSE TITLE: LOGICAL FUNCTIONS

EXAM VENUE: **STREAM: BSc Y1S1**

TIME: 2 HOURS **EXAM SESSION:**

Instructions:

Answer question1 and any other two questions

1. **Show all the necessary working**
2. **Candidates are advised not to write on the question paper**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

QUESTION 1 (30 MARKS)

(a) Explain the following terms as used in Set theory (4 marks)
- (i) Subset
- (ii) Union of sets
- (iii) Intersection of sets
- (iv) Null set

(b) Determine the power set P(A) of $A = \{a, b, c, d\}$ (4 marks)

(c) Prove that $\dfrac{\sin^2\theta - 3\cos^2\theta + 1}{\sin^2\theta - \cos^2\theta} \equiv 2$ (6 marks)

(d) Convert each of the following binary numbers to their decimal equivalents.
- (i) 101010 (3 marks)
- (ii) 10011.10011 (4 marks)

(e) Let p be "it is cold" and let q be "it is raining". Write a simple sentence which describe each of the following statements. (4 marks)
- (i) $-p$
- (ii) $p \wedge q$
- (iii) $p \vee q$
- (iv) $q \vee -p$

(f) Construct the truth table of $-(p \wedge -q)$ (5 marks)

QUESTION 2 (20 MARKS)

(a) Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$, $B = \{1, 3, 4, 5, 7\}$ and $C = \{7, 8\}$. Find:
- (i) $A - C$ (1 mark)
- (ii) $B^c \cap C$ (2 marks)
- (iii) $\left((A \cup B) \cap C^c\right)^c$ (4 marks)

(b) Prove the following distributive law of set operation. (4 marks)
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(c) On a standard three-circle Venn diagram:
- (i) Shade the regions corresponding to the set expression
  $(P^C \cap Q) \cup (P \cap R)$ (4 marks)
- (ii) Show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (5 marks)

QUESTION 3 (20 MARKS)

(a) Verify that:

(i) $$\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)} = \frac{1-\tan\alpha\tan\beta}{1+\tan\alpha\tan\beta}$$ (5 marks)

(ii) $\cos 2\alpha = 1-2\sin^2\alpha$ (3 marks)

(b) Solve the equation $15\cos^2 x + 7\cos x - 2 = 0$ for $0 \le x \le 2\pi^c$ (6 marks)

(c) Taking $15 = 60 - 45$, find the value of the sine and the cosine of 15^0 (6 marks)

QUESTION 4 (20 MARKS)

(a) Convert each of the following decimal numbers to their binary equivalents:

(i) 87 (4 marks)

(ii) 34.75 (4 marks)

(b) Convert :

(i) $A3F.C_{16}$ to decimal equivalent. (4 marks)

(ii) 250.25_{10} to hexadecimal equivalent. (4 marks)

(c) Solve the following binary arithmetic problems:

(i) $$\begin{array}{r} 1111 \\ \underline{+111} \end{array}$$ (2 marks)

(ii) $$\begin{array}{r} 10001 \\ \underline{-\ \ 110} \end{array}$$ (2 marks)

QUESTION 5 (20 MARKS)

(a) Let p denote "He is very rich" and let q denote "He is happy". Write each of the following statement in symbolic form using p and q. (Note that "He is poor" and "He is unhappy" are equivalent to $-p$ *and* $-q$ respectively). (4 marks)

(i) If he is rich, then he is unhappy.
(ii) He is neither rich nor happy.
(iii) It is necessary to be poor in order to be happy.
(iv) To be poor is to be happy

(b) Using a truth table, verify that:

(i) $p \vee -(p \wedge q)$ is a tautology. (4 marks)
(ii) $(p \wedge q) \wedge -(p \vee q)$ is a contradiction. (4 marks)

(c) Let a, b be any element in a Boolean algebra **B** . Prove that:

(i) $a * a = a$ (2 marks)

(ii) $a * (a + b) = a$ (2 marks)

(d) Given that the set D_m of divisors of m is a bounded, distributive lattice with $a + b = a \vee b = lcm(a,b)$ and $a * b = a \wedge b = \gcd(a,b)$. Show that D_m is a Boolean algebra if m is square free. (4 marks)