JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITYDRAFT EXAMINATION FOR BSc/BEd IN MATHEMATICS

2nd YEAR 1st SEMESTER 2017/2018 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SMA201
COURSE TITLE: LINEAR ALGEBRA II

EXAM VENUE: **STREAM: BSc Y4S1**

TIME: 2 HOURS   **EXAM SESSION:**

Instructions:

Answer question1 and any other two questions
1. **Show all the necessary working**
2. **Candidates are advised not to write on the question paper**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

.

QUESTION ONE [30 MARKS] COMPULSORY

a) Define the following terms in relation to linear spaces
    i)   Span   (2mk)
    ii)   Dimension   (2mk)

b) State the eigenvalues of the matrix $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ (6mks)

c) Let $T: \mathbf{R}^3 \to \mathbf{R}^3$ be a linear operator defined by
    $T(x_1, x_{2,} x_3) = (2x_1 - x_2 - x_3, x_1 - x_3, -x_1 + x_2 + 2x_3)$. Find the matrix B
    associated with T with respect to the standard ordered basis   (6mks)

d) Let $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$.
    i) Find all the eigenvalues of A and the corresponding eigenvectors   (6mks)
    ii) Find matrix P which diagonalizes A.   (8mks)

QUESTION TWO (20 MARKS)

a) Let $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 6 \\ -2 & 4 \end{bmatrix}$. Show that $(AB)^{-1} = B^{-1}A^{-1}$ (6mks)

b) Let $A = \begin{bmatrix} 1+i & 4 \\ 1-i & -6 \end{bmatrix}$. Find the
    i) Conjugate of matrix A (2mk)

    ii) Conjugate transpose of matrix A (2mk)

c) Consider the following bases $B = \{(1,0), (0,1)\}$ and $B' = \{(1,2), (2,3)\}$ for $\mathbf{R}^2$. If
    $T: \mathbf{R}^2 \to R^2$ is a linear transposition defined by $T(x_1, x_2) = (x_1 + 7x_2, 3x_1 - 4x_2)$.
    i) Find A, the matrix of representation of T with respect to B
    ii) Find M, the matrix of representation of T with respect to B' (4mks)
    iii) Find N, the transition matrix of representation of T with respect B to B'
    (4mks)

QUESTION THREE (20 MARKS)

Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be a linear operator from a vector space $\mathbf{R}^3$ to itself defined

by $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_2 - x_3 \\ x_1 - x_3 \\ -x_1 + x_2 + 2x_3 \end{bmatrix}$

    i) Obtain M the matrix of the linear operator T (8mks)

    ii) Find the characteristic polynomial of the operator T (6mks)

    iii) Find the eigenvalues of T and their corresponding eigenvectors   (6mks)

QUESTION FOUR (20 MARKS)

a) Verify that the set $S = \{(x,\ 3x): x \in \mathbf{R}\}$ is a subspace of $\mathbf{R}^2$ (4mks)

b) i) State Cayley Hamilton theorem   (3mks)

    ii) Give matrix $A = \begin{bmatrix} 2 & 5 \\ 1 & -3 \end{bmatrix}$,show that both A , A^{-1} satisfy Cayley Hamilton theorem .

    (8mks)

c) i) Define the term kernel   (2mk)

    ii) If $T: U \to V$ is a linear mapping, show that the kernel of T is a subspace of U
    (3mks)

d) Find the matrix of linear mapping $T: P_3 \to P_1$ given by $T(f) = f'' + f'''$ (4mks)

QUESTION FIVE (20 MARKS)

a) Define the term orthogonality of vectors in a vector space W (4mks)

b) Let $P = \begin{pmatrix} 1 & -2 & 2 \\ 2 & 2 & 1 \\ -2 & 1 & 2 \end{pmatrix}$ be a real square matrix.

Prove that P is orthogonal hence and find $\hat{P}$ the orthonormalized form of P and $\hat{P}^{-1}$
.**[10marks]**

c) If V is a linear space of all functions of the form $f(t) = c_1 \cos t + c_2 \sin t$, where c_1 and c_2 are arbitrary constants,
Find the matrix of linear transformation $T(f) = f''' + af'' + bf'$ with respect to the basis $\cos t$, $\sin t$ where a and b are arbitrary constants   (6mks)