JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE

1^{ST} YEAR 1^{ST} SEMESTER 2018/2019 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SMA 3111

COURSE TITLE: CALCULUS I

EXAM VENUE: **STREAM: (………………)**

DATE: …………………… **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY) (30 marks)

a) Let $f(x)$ be a function. Define a continuous function $f(x)$ at $x = x_0$ (3 marks)

b) Find $\lim_{x \to 2} \frac{x^3 + x^2 - 2x - 8}{x - 2}$ (5 marks)

c) Given

$$f(x) = \begin{cases} 1/x, & x > 0 \\ 3x + 2, & x < 0 \end{cases}$$

Find the one-sided limits: (4 marks)

$\lim_{x \to 0+} f(x)$ $\lim_{x \to 0-} f(x)$ $\lim_{x \to +\infty} f(x)$ $\lim_{x \to -\infty} f(x)$

d) Determine the point of discontinuity (if any) of the function $f(x)$

$$f(x) = \frac{x - 1}{(x + 3)(x - 2)}$$

State the type of discontinuity at the points. (4 marks)

e) Given that $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, find $f'(x)$ if $f(x) = x^2 - 2x$, and hence find the value of the derivative: $f'(-3)$ (5 marks)

f) Find the first and second derivatives of the function below:
$y = 6\cos 2x - 10e^{3x} - \frac{5}{x^2}$ (4 marks)

g) Find $\frac{dy}{dx}$ by implicit differentiation, if $x^2 y^2 + x \sin y = 4$. (4 marks)

h) Given that $f(x) = \frac{2x+1}{x^2 - 1}$, find $f'(x)$ (4 marks)

QUESTION TWO (20 marks)

a) Evaluate $\frac{dy}{dx}$ at $x = 2.5$, correct to 4 significant figures, given $y = \frac{2x^2 + 3}{\ln 2x}$. (5 marks)

b) Evaluate $\lim_{x \to \infty} \frac{4x^4 + 5}{\left(x^2 - 2\right)\left(2x^2 - 1\right)}$. (5 marks)

c) Find all the critical numbers of $f(x) = x^3 - 5x^2 - 8x + 3$ (4 marks)

d) If $3x^2 + 2x^2 y^3 - \frac{5}{4} y^2 = 0$ evaluate $\frac{dy}{dx}$ when $x = \frac{1}{2}$ and $y = 1$. (6 marks)

QUESTION THREE (20 marks)

a) Find $D_x f(x)$ given

$f(x) = e^{2t} \ln 3t$ (5 marks)

b) Discuss the continuity of the function $f(x)$ given that;

$$f(x) = \begin{cases} x + 2, & -1 \leq x \leq 3 \\ 14 - x^2, & 3 \leq x \leq 5 \end{cases}$$

(5 marks)

c) Differentiate $y = tan^2(3x - 2)$ with respect to x. (5 marks)

d) Prove that $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

(5 marks)

QUESTION FOUR (20 marks)

a) If $y = 3^{x^2+x}$, find the first derivative (4 marks)

b) Find $\frac{dy}{dx}$ given that $y = \sin 3x + \cos 2x$ (5 marks)

c) A point moves along the curve $y = x^3 - 3x + 5$ so that $x = \frac{1}{2}\sqrt{t} + 3$ here t is time. At hat rate is y changing hen $t = 4$ (5 marks)

d) Compute the folloing deriatives (6 marks)

i. $y = \ln(x \sin x + 1)$

ii. $y = e^{x^2}$

QUESTION FIVE (20 marks)

a) If $x = 2t/(t+2)$, $y = 3t/(t+3)$, find $\frac{dy}{dx}$ in terms of t. (5 marks)

b) The displacement s cm of the end of a stiff string at time t seconds is given by: $s = ae^{-kt} \sin 2\pi ft$. Determine the velocity and acceleration of the end of the spring after 2 seconds if $a = 3$, $k = 0.75$ and $f = 20$. (5 marks)

c) Determine for the curve $y = 2x^2 - 3x$ at the point $(2,2)$ the equation of the normal. (5 marks)

d) Calculate the derivate of $\sqrt{7x^3 - 2x^2 + 5}$ (5 marks)