JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY DRAFT EXAMINATION FOR MSC IN APPLIED MATHEMATICS

1st YEAR 1st SEMESTER 2017/2018 ACADEMIC YEAR

KISUMU CAMPUS

COURSE CODE: SMA 807

COURSE TITLE: COMPLEX ANALYSIS I

EXAM VENUE: **STREAM: MSc Y1S1**

TIME: 3 HOURS **EXAM SESSION:**

Instructions:

Answer any three questions

1. **Show all the necessary working**
2. **Candidates are advised not to write on the question paper**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

Question1 [20 marks]

(a) Express $(1)^{1/6}$ in rational Cartesian form. [6marks]

(b) .Let D be a rectangular region bounded by lines $x=0$, $y=0$, $x=2$ *and* $y=1$.

Define the mapping $\omega(z)=(12+i)z+(1+2i)$ on D into D'.

(i) Show that ω is a conformal mapping. **(ii)** Obtain the translation , rotation and dilation factor , of D into D'

[5marks]

(c) Classify the singularities of the complex function.

(i) $f(z)=\dfrac{1}{z-i}-\dfrac{1}{z}$ (ii) $f(z)=\dfrac{\sin z}{z}$, (iii) $f(z)=z^{3/2}$ (iv) $f(z)=\dfrac{z}{\sin z}$ [4marks]

(d) Suppose $f(z)=z^3$ and $\Delta z=z-z_0$, determine the $\lim\limits_{\Delta z\to 0}\left\{\dfrac{f(z)-f(z_0)}{\Delta z}\right\}$

and hence find $f'(z_0)$. [5 marks]

Question 2 [20 marks]

(a) If $f(z)=z\overline{z}$ find $\lim\limits_{z\to z_0}\left\{\dfrac{f(z)-f(z_0)}{z-z_0}\right\}$. Discuss the existence $f'(z_0)$, the derivative of $f(z)$ on the complex plane. [6marks]

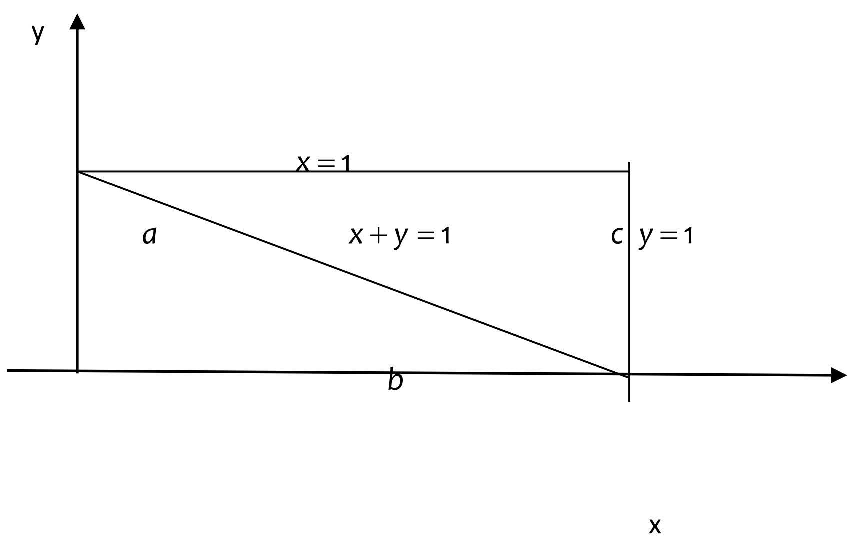
(b) Find all the points at which the function $f(z)=x^2-y^2+x\ +i(2xy-y)$ is analytic. [4 marks]

(c) Evaluate the integral : $\oint_{|z|=3}\dfrac{z}{(z^2-9)^3}dz$ [6 marks]

(d) Prove that $u=e^{-x}(x\sin y-y\cos y)$ is harmonic. [4 marks]

Question 3 [20 marks]

Let D be the triangular region bounded by lines $x=1$, $y=1$ *and* $x+y=1$ as shown figure 1 below. Find D' the image of D under the mapping $\omega(z)=z^2+(1+i)$.



z **-PLANE** Fig.1 [14 marks]

Determine explicitly the equations governing the arc lengths of D' **.**
Give the coordinates of D' and sketch D' on the $u-v$ plane. [6 marks

Question 4 [20 marks]

(a) Evaluate the integral $\oint_C z^2 dz : C$ *is the curve* $y=\dfrac{1}{x^2}$ *from* $z=1+i$ *to* $z=3+\dfrac{i}{19}$. [8 marks]

(b) Suppose that a function f is analytic in a star D . Suppose further that C is a closed contour lying in D . Prove that $\oint_C f(z)dz=0$. **[5marks]**

(c) Determine the value of the contour integral $\oint_{|z|=3} \dfrac{e^z+\sin z}{z^2-25}dz$ where the contour of integration is the circle centre at 0 and with radius 3 followed in the positive (anticlockwise) direction.
[7 marks]

Question 5 [20 marks]

(a) (i)State and prove Rouche's theorem. [4marks]

(ii)Determine the number of $G(z)=e^{z}-4z^{2}+8z-0.1$ [2marks]

(b)Determine the value of the contour integral $\oint_{|z|=10} \dfrac{e^{tz}}{z^{2}(z-10)(z^{2}+2z+2)}dz$ where the contour of integration is the circle centre at 0 and radius 10 followed in the positive (anticlockwise) direction.

[10marks]

(c) Evaluate the improper integral $I = \int_{0}^{\infty} \dfrac{\log^{2} x}{x^{2}-1}dx$. **[4 marks]**