



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE

ACTUARIAL

2ND YEAR 1ST SEMESTER 2018/2019 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SAC 203

COURSE TITLE: FUNDAMENTALS OF ACTUARIAL MATHEMATICS II

EXAM VENUE:

STREAM: (BSc. Actuarial)

DATE:

EXAM SESSION:

TIME: 2.00 HOURS

Instructions:

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

- (a) Explain the following concepts as applied in Actuarial Studies.
- Annuity certain payable in advance.
 - Cash flow
 - Insurance lapse
 - Guaranteed Endowment
 - Force of mortality. [5 marks]

- (b) Suppose it is given that $A_x = 0.25$, $A_{x+20} = 0.4$, $A_{x:\overline{20}|} = 0.55$, and $i = 0.03$. Assume deaths are uniformly distributed over each year of age, where the symbols have their usual meanings as used in Actuarial Mathematics. Calculate $1000\bar{A}_{x:\overline{20}|}$. [5 mks]

- (c) Show that;

$${}_n a_x = v^n {}_n p_x a_{x+n}$$

[4 marks]

- (d) Show that if Z_1 and Z_2 are the present value random variables for an n - year term and n - year pure endowment, then $Z_1 \cdot Z_2 = 0$. [3 marks]

- (e) Calculate \ddot{a}_{70} , given that $A_{50:\overline{20}|} = 0.42247$, $A_{50:\overline{20}|}^1 = 0.14996$, $A_{50} = 0.31266$. [5 mks]

- (f) For a whole life insurance with a sum assured of Ksh 150000 paid at the end of the year of death, issued to (x) , you are given ${}^2A_x = 0.0143$, $A_x = 0.0653$ and the annual premium is determined using the equivalence principle. Calculate the standard deviation of L_0^n . [3 marks]

- (g) An investor wins a prize which can be taken either as a lump sum payment of 10000 dollars paid immediately or as monthly payments of Ksh 100 per month paid in advance for life.

The investor is aged 60 and his mortality is given in the AM92 table.

Find the form of prize which is preferable and justify your answers. [5 marks]

QUESTION TWO

- (a) Explain the concept of pure endowment factor and derive its expected present value indicating who would wish to buy such a policy and explain why this benefit may be unpopular. [4 marks]

- (b) Show that

$$A_{x:\overline{1}|} = v.$$

[4 marks]

- (c) For a return of 5000 dollars in 4.5 years' time, how much do you have to invest today at an effective interest rate of 5 percent per annum? [4 marks]

- (d) Consider an n - year term endowment policy such that;

- it provides for a benefit either on the death of (x) or on survival of (x) to the end of the n - year term whichever event occurs first.

- the death benefit is payable at the end of the year of death.
 - the death and survival benefits are both of 1 unit of money.
- i. Let Z_1 and Z_2 be the present values of the death and survival benefits correspondingly, write these two random values in terms of $T(x)$, and hence find their probability mass functions. [5 marks]
 - ii. Let $A_{x:\overline{n}}$ be the expected present value of the benefits under this policy, so that

$$A_{x:\overline{n}} = E(Z_1) + E(Z_2)$$

show that

$$A_{x:\overline{n}} = 1 - da_{x:\overline{n}}$$

[3 marks]

QUESTION THREE

- (a) On the basis of AM92 mortality table and 4% interest. Calculate the APV's of each of the following assurance benefits for a life aged 30 using commutation functions;
 - i. a whole life insurance with Ksh 10000 payable immediately on death. [3 marks]
 - ii. a 20 - year term insurance with Ksh 50000 payable at the end of the year of death. [3 marks]
 - iii. a 20 year endowment insurance with a sum assured of Ksh 50000 with death benefit payable immediately on death. [3 marks]
 - iv. a deferred temporary assurance with a sum assured of Ksh 100000 payable at the end of the year of death if death occurs between ages 40 and 50 exactly. [4 marks]
- (b) A life aged 50 is subject to mortality of AM92 table, effects a pure endowment policy with a term of 20 years for a sum assured of Ksh 10000.
 - i. Write down the PVRV of the benefits under this contract. [2 marks]
 - ii. Assuming an effective interest rate of 5%, calculate the mean and variance of this PVRV. [5 marks]

QUESTION FOUR

- (a) A special 3 year endowment insurance policy is issued to (50) where the death benefit of 200 is payable at the end of the year of death, the endowment of 500 is payable at maturity. Assuming a survival model of

$$s(x) = \frac{1}{200}(110 - x)^{\frac{2}{3}}, \quad 0 \leq x \leq 110.$$

Calculate the APV of this policy if interest rate is assumed to be 5%. [8 marks]

- (b) Given the following table of values for l_x and A_x , assuming an interest rate of 6% per year.

x	l_x	A_x
35	100000	0.151375
36	99737.15	0.158245
37	99455.91	0.165386
38	99154.72	0.172804
39	98831.91	0.180505
40	98484.68	0.188492

Calculate

i. $v^5 {}_5p_{35}$. [2 marks]

ii. $A_{35:\overline{35}|}^1$. [2 marks]

iii. ${}_5|A_{35}$. [2 marks]

iv. $\bar{A}_{35:\overline{3}|}$ assuming a UDD. [3 marks]

- (c) Consider an n - year temporary life annuity due with monthly payments at a rate of 1 p.a for a life age x now, that is , a level annuity-due, contingent on the survival of (x) , payable monthly in advance at a rate of 1 p.a. for at most n years (the maximum number of payments possible is $12n$) . Denote by $\ddot{a}_{x:\overline{n}|}^{(12)}$ its expected present value. Show that

$$\ddot{a}_{x:\overline{n}|}^{(12)} = \frac{1}{12} \sum_{j=0}^{12n-1} \frac{D_{x+\frac{j}{12}}}{D_x}$$

[3 marks]

QUESTION FIVE

For a group of 200 lives with exact age (x) , and independent future lifetimes, given that each life is to be paid 1 at the beginning of each year if alive, $A_x = 0.2$, ${}^2A_x = 0.06$. $i = 0.04$. Y_i is the PVRV of the payments for life $i = 1, 2, \dots, 200$, so that the present value of the aggregate payments is the sum of these that is

$$Y = \sum_{i=1}^{200} Y_i.$$

- (a) Calculate the mean of Y and hence its variance. [10 marks]

- (b) Using the Normal approximation, calculate the initial size of the fund needed in order to be 95% certain of being able to make payments for these life annuities. [10 marks]