JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE

2018/2019 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SAS 201

COURSE TITLE: SAMPLE SURVEY

EXAM VENUE: **STREAM: ACTUARIAL SCIENCE**

DATE: **EXAM SESSION: DECEMBER 2018**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question One Compulsory (30mks)

i) Carefully define a simple random sample of size n from a population of size N (4marks)

ii) List three reasons for choosing to use stratified random sampling rather than simple random sampling. (3marks)

iii) Carefully discuss two situations in which cluster sampling is an optimal design for obtaining a specified amount of information. (5marks)

iv) A population consists of $N = 5$ observations: $X_i = 3, 9, 6, 12, 15$. One wishes to obtain simple random samples of size $n = 3$. Obtain all possible simple random samples such that $\hat{V}(\bar{x}) = \frac{N-n}{Nn}S^2$ and verify that $\bar{x}$ is unbiased for $\bar{X}$ (6marks)

v) Explain the following terms as used in sample surveys:

a) Sampling unit

b) Sampling frame

c) Purposive sample

d) Simple Random Sampling without Replacement. (8 marks)

vi) A population of 1000 is divided into 4 strata. The sizes of the strata and variances are given as follows.

strata	A	B	C	D
size	180	250	270	300
variance	25	64	121	169

A stratified sample of size 100 is to be drawn from the population. Determine the sample sizes in case of

a) Proportional allocation.

b) Optimum allocation (4marks)

Question Two (20mks)

The results from a simple random sampling from a stratified population are given in the following table. The columns are the strata sizes, the strata sample sizes, the sample means and the sample variances.

Stratum	N_h	n_h	$\bar{y}_h$	s_h^2
1	100	20	115.7	57.6
2	300	20	147.2	46.9
3	400	30	133.6	75.3

i) Find a 95% confidence interval for the population mean.

ii) The person taking the sampled claimed that they use optimal allocation to select the sample size based on good information about the likely sizes of the strata variances. Given their allocation can you determine their prior guess for the strata variances? Explain. Based on the results of the sample does it appear that they made good choices with their guesses for the strata variances. Assuming the observed sample variances are the true sample variances find the optimal allocation for a sample size of 70

Question Three (20mks)

We are interested in estimating the proportion of men P affected by an occupational sickness in a business of 1500 workers. In addition, we know that three out of 10 workers are usually affected by this sickness in businesses of the same type. We propose to select a sample by means of a simple random sample.

i) What sample size must be selected so that the total length of a confidence interval with a 0.95 confidence level is less than 0.02 for simple designs with replacement and without replacement?

ii) What should we do if we do not know the proportion of men usually affected by the sickness (for the case of a design without replacement)? To avoid confusions in notation, we will use the subscript WR for estimators with replacement, and the subscript WOR for estimators without replacement.

Question Four (20mks)

A sample of 100 students is chosen using a simple random design without replacement from a population of 1000 students. We are then interested in the results obtained by these students in an exam. There are two possible results: success or failure. The outcome is presented in the Table below

	Men	Total	Total
Success	$n_{11} = 35$	$n_{12} = 25$	$n_{1.} = 60$
Failure	$n_{21} = 20$	$n_{22} = 20$	$n2. = 40$
	$n_{.1} = 55$	$n_{.2} = 45$	$n = 100$

i) Estimate the success rate for men and for women.

ii) Calculate the approximate bias of the estimated success rates.

iii) Estimate the mean square error of these success rates.

iv) Give the 95% confidence intervals for the success rate for men R_M and for women R_W. What can we say about their respective positions?

Question Five (20mks)

i) What sample size is needed if we choose a simple random sample to find, within two percentage points (at least) and with 95 chances out of 100, the proportion of Parisians that wear glasses? (10marks)

ii) Let $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ be variance based on sample observations of x_i. Show that under SRSWOR, $E(s^2) = S^2$ where $S^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2$ (10marks)

iii) We want to estimate the surface area cultivated on the farms of a rural township. Of the N = 2010 farms that comprise the township, we select 100 using simple random sampling. We measure yk, the surface area cultivated on the farm k in hectares, and we find ∑yk=2907ha and ∑yk2 = 154593ha2

Give the value of the standard unbiased estimator of the mean

$$\bar{Y} = \frac{1}{N}\sum_{k \in S} y_k$$