JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION SCIENCE/BACHELOR OF SCIENCE(ACTUARIAL SCIENCE WITH IT)

2ND YEAR 1ST SEMESTER 2018/2019 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SMA 200
COURSE TITLE: CALCULUS I

EXAM VENUE: **STREAM:**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. Answer question one (compulsory) and any other two questions.

2. Candidates are advised not to write on the question paper.

3. Candidates must hand in their answer booklets to the invigilator while in the examination room.

QUESTION ONE (COMPULSORY 30 MARKS)

a) Evaluate $\int (x^2 + 3x + 2)dx$ (3 marks)

b) By using an appropriate technique evaluate $\int \sec x\, dx$ (4 marks)

c) Use the product rule for the derivative of the product uv to show that

$\int v\,du = uv - \int u\,dv$ hence evaluate the definite integral $\int_{1}^{e} x^2 \ln x\, dx$.   (7 marks)

d) Show that $\int \dfrac{dx}{a^2 + x^2} = \dfrac{1}{a} \tan^{-1} \dfrac{x}{a}$ (6 marks)

e) Use a trigonometric substituition to evaluate product $\int \dfrac{1}{x^2 \sqrt{16 - x^2}} dx$

                                                         (6 marks)

f) Eva luate the integral $\int \sqrt{1 + \cos 4x}\, dx$ (4 marks)

QUESTION TWO (20 MARKS)

a) Show that $\int \cos^n x = \dfrac{\cos^{n-1} x \sin x}{n} + \dfrac{n-1}{n} \int \cos^{n-2} x\, dx$, hence or otherwise evaluate

$\int \cos^3 x\, dx$ (9 marks)

b) By rewriting the denominator evaluate $\int \dfrac{1}{\sqrt{1 + 8x - 4x^2}} dx$ (6 marks)

c) Use partial fractions to rewrite $\dfrac{x-11}{(x+5)(x+4)}$ hence evaluate $\int \dfrac{x-11}{(x+5)(x+4)} dx$

                                                       (5 marks)

QUESTION THREE (20 MARKS)

a) Find the Taylor series generated by $f(x) = \dfrac{1}{x}$ at $a = 2$.   (7 marks)

b) Use Simpson's rule to estimate the length of an arc $L = \int_{0}^{\pi} \sqrt{1 + \cos^2 x}\, dx$,

with $n = 8$.   (6 marks)

c) Find the power series for the function $f(x) = \ln(1 + 2x)$ (7 marks)

QUESTION FOUR (20 MARKS)

a) Sketch the curve $y = x^2 - x - 2$ from $x = -1$ to $x = 3$, hence find the area enclosed by the curve the straight lines and the x-axis.   (8 marks)

b) The region bounded by the curve $y = 9 - x^2$ and the line $y = 3 - x$ is rotated about the x-axis to generate a solid. Find the volume of the solid.

                                                       (6 marks)

c) Determine the lateral surface area of a cone generated by revolving a line segment $x = 2y + 4$, $0 \le x \le 2$ about the y-axis. (6 marks)

QUESTION FIVE (20 MARKS)

a) Evaluate $\int e^{ax} \sin bx \, dx$ (7 marks)

b) Determine whether the series $\sum_{n=1}^{\infty} \dfrac{n^2(n+1)!}{n^2!}$ is convergent or divergent.

(6 marks)

c) Evaluate the improper integral $\displaystyle\int_{-\infty}^{\infty} \dfrac{2x}{\left(x^2+1\right)^2} \, dx$ (7 marks)