



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE

ACTUARIAL

3RD YEAR 1ST SEMESTER 2018/2019 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SAC 301

COURSE TITLE: METHODS OF ACTUARIAL INVESTIGATIONS I

EXAM VENUE:

STREAM: (BSc. Actuarial)

DATE:

EXAM SESSION:

TIME: 2.00 HOURS

Instructions:

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

- (a) Define the following terms as used in Financial Mathematics.
- Immunisation.
 - Convexity.
 - Spot rate.
 - Property.
 - Macaulay Duration. [5 marks]
- (b) An investor purchases 2,000 nominal of a bond redeemable at 110% in exactly six year's time. Coupons at a rate of 8% pa are payable half yearly in arrear. Calculate the amount of each coupon and the redemption amount. [3 marks]
- (c) A ten-year bond with half yearly coupons of 6% pa has just been issued with a redemption yield of 9% pa effective. It is redeemable at par. What price would an investor paying 15% tax on income pay for the bond? Tax payments are due four months after each coupon is received. [5 marks]
- (d) Calculate, using 10% pa interest, the convexity of the following assets, each of which has a discounted mean term of 11 years. Comment on your answers.
- Asset A is an 11-year zero coupon bond.
 - Asset B will provide a lump sum payment of 9,663 in 5 year's time and a lump sum payment of 26,910 in 20 years time.
 - Asset C is a level annuity of 1 pa payable annually in arrears for 50 years. [9 marks]
- (e) If the n year spot rates can be approximated by the function $0.06 - 0.03e^{-0.1n}$, calculate the one-year forward rate at time 10. [2 marks]
- (f) A fund must make payments of 50,000 at the end of the sixth and eighth years. Show that, if interest rates are currently 7% p.a at all durations, immunisation to small changes in interest rates can be achieved by holding an appropriately chosen combination of a 5- year zero-coupon bond and a 10-year zero-coupon bond. [6 marks]

QUESTION TWO

- (a) What happens to yields of fixed interest securities if:
- Bond prices fall
 - Demand for fixed-interest securities falls
 - The government issues many more stocks
 - Institutional investors suddenly decide to invest less in equities and more in fixed- interest securities
 - Bond prices rise. [10 marks]
- (b) The current annual term structure of interest rates is:

6%, 6%, 6%, 6%, 7%

Calculate the gross redemption yield of a five-year fixed interest security redeemable at par if the annual coupon is

- 2%
- 4%. [4 marks]

- (c) Evaluate the discounted mean term of a bond redeemable at par in 10 years time with annual coupons of 8% at interest rates of 5%, 10% and 15%. Hence sketch a graph of the discounted mean term as a function of the interest rate over the range 5% to 15%. [6 marks]

QUESTION THREE

- (a) In a particular bond market, the two-year par yield at time $t = 0$ is 4.15% and the issue price at time $t = 0$ of a two-year fixed interest stock, paying coupons of 8% annually in arrears and redeemed at 98, is 105.40 per 100 nominal. Calculate:
- the one-year spot rate
 - the two-year spot rate. [6 marks]
- (b) At 1 July 2004, an investor has a liability of 20,000 to be paid on 1 January 2008 and a liability of 18,000 to be paid on 1 July 2010. The investor currently holds assets with a present value equal to the present value of the liabilities. The investor wishes to immunise its position by investing in two zero coupon bonds with outstanding terms of four years and seven years. Determine whether or not this is possible assuming an effective interest rate of 10% per annum. [6 marks]
- (c) An index-linked bond pays half-yearly coupons and is redeemable at par on 28 February 2024. The coupon paid on 28 February 2008 was 2.10. A non-taxpayer buys 100 nominal of the bond on 29 February 2008. Assuming that future inflation is 5.25% p.a, how much should this investor pay in order to obtain a money rate of return of 10% p.a.? [5 marks]
- (d) Ten years ago, a saver invested 5,000 in an investment fund operated by an insurance company. Over this period the rate of return earned by the fund has averaged 12% per annum. If prices have increased by 80% over this period, calculate the average annual real rate of interest earned by the fund over this period. [3 marks]

QUESTION FOUR

- (a) The price at time 0, P_t of a zero coupon bond for $2 < t < 4$ is given by the equation

$$P_t = (100 - 2t)\%.$$

Calculate the instantaneous forward rate F_3 . [3 marks]

- (b) An investor purchases a bond 3 months after issue. The bond will be redeemed at par ten years after issue and pays coupons of 6% per annum annually in arrears. The investor pays tax of 25% on both income and capital gains (with no relief for indexation).
- Calculate the purchase price of the bond per 100 nominal to provide the investor with a rate of return of 8% per annum effective. [6 marks]
 - The real rate of return expected by the investor from the bond is 3% per annum effective. Calculate the annual rate of inflation expected by the investor. [1mark]
- (c) For the last 10 years a man has paid 50 at the start of each month into a savings account that has achieved a real rate of interest of 3% per annum over this period. If inflation has been at a constant rate of 5% per annum, calculate the balance of the man's account today. [5 marks]
- (d) A fixed interest stock with a coupon of 8% per annum payable half yearly in arrears can be redeemed at the option of the lender (i.e the investor) at any time between 10 and 15 years from the date of issue. What price should an investor subject to tax at 25% on income, who wishes to obtain a net yield of at least 7% per annum, pay for 10000 nominal of this stock?

QUESTION FIVE

- (a) An investor purchases a bond 3 months after issue. The bond will be redeemed at par ten years after issue and pays coupons of 6% per annum annually in arrears. The investor pays tax of 25% on both income and capital gains (with no relief for indexation).
- Calculate the purchase price of the bond per 100 nominal to provide the investor with a rate of return of 8% per annum effective. [6 marks]
 - The real rate of return expected by the investor from the bond is 3% per annum effective. Calculate the annual rate of inflation expected by the investor. [2 marks]
- (b) Find the upper and lower quartiles for the accumulated value at the end of 5 years of an initial investment of 1,000, using the varying interest rate model and assuming that the annual growth rate has a log-normal distribution with parameters $\mu = 0.075$ and $\sigma^2 = 0.0252$. [7 marks]
- (c) An investor, who is liable to income tax at 20% but is not liable to capital gains tax, wishes to earn a net effective rate of return of 5% per annum. A bond bearing coupons payable half-yearly in arrear at a rate 6.25% per annum is available. The bond will be redeemed at par on a coupon date between 10 and 15 years after the date of issue, inclusive. The date of redemption is at the option of the borrower. Calculate the maximum price that the investor is willing to pay for the bond. [5 marks]