JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION

ARTS, SPECIAL EDUCATION AND EDUCATION SCIENCE

3RD YEAR 1ST 2018/2019 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SAS 309

COURSE TITLE: TIME SERIES ANALYSIS AND FORECASTING

EXAM VENUE: **STREAM: (B.sc ACTUARIAL SCIENCE)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) Explain FOUR types of variations in time series analysis. (4 Marks)

b) Consider a time series $X_t = e_1 \cos \lambda t + e_2 \sin \lambda t$ where e_1 and e_2 are independent random variables that are normally distributed with mean zero and variance σ^2. Show that X_t is second order stationary. (4 Marks)

c) From the data given in the table below on average monthly production of coal in millions of kilograms , construct;

i. a 5 year moving average (2 Marks)

ii. a 4 year moving average (2 Marks)

iii. a 4 year centered moving average (2 Marks)

Year	1948	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958
Average monthly production of coal	50.0	36.5	43.0	44.5	38.9	38.1	32.6	38.7	41.7	41.1	33.8

d) Let μ_t be white noise, where

$$E(\mu_t) = 0 \text{ for all } t$$

$$E(\mu_t^2) = 20 \text{ for all } t$$

$$E(\mu_t \mu_{t-s}) = 0 \text{ for all } t \text{ and } s \text{ where } s \neq 0$$

Let $y_t = \mu_t + 0.7\mu_{t-1} + 0.1\mu_{t-2}$. Determine the numerical values of

i. $\text{var}(y_t)$

ii. the correlation between y_t and y_{t-1}

iii. the covariance between y_t and y_{t-1}

e) The data below gives the average quarterly prices of a commodity for four years, calculate the quarterly indices using multiplicative model. (4 Marks)

YEAR	I	II	II	IV
2009	40.3	44.8	46.0	48.0
2010	30.1	53.1	55.3	59.5
2011	47.2	54.1	52.1	55.2
2012	55.4	59.0	61.6	65.3

f) Consider the following MA(1) process

$$A: X_t = e_t + \theta e_{t-1} \qquad \theta > 0$$

$$B: X_t = e_t + \frac{1}{\theta} e_{t-1}$$

Show that $\rho(h)=\begin{cases} 1 & h=0 \\ \dfrac{\theta}{1+\theta^2} & h=\pm 1 \\ 0 & h>1 \end{cases}$ for both process. (6 Marks)

QUESTION TWO (20 MARKS)

Consider the following data

t	1	2	3	4	5	6	7	8
X_t	1	2	3	7	5	8	5	9

Determine;

i. the first four sample autcovariance.

ii. the sample autocorrelations at lags 0, 1, 2, 3 and 4

iii. the standard errors for the sample autocorrelations at lags 1, 2, 3 and 4.

QUESTION THREE (20 MARKS)

a) Using the table below for production of sugar in 1000 tonnes from Mumias Sugar Company (16 Marks)

Year	1963	1965	1966	1967	1968	1969	1972
Production	77	98	104	98	81	98	90

i. Fit trend line equation and tabulate the trend values.

ii. Assuming an additive model eliminate trend. What components of time series are left over?

iii. What is the monthly increase in the production of sugar?

iv. Estimate the production of sugar in 1970.

b) Explain FOUR merits of time series. (4 Marks)

QUESTION FOUR (20 MARKS)

Given three selected point X_1, X_2 and X_3 corresponds to $t_1=2$, $t_2=30$, $t_3=58$, $X_1=58.8$, $X_2=138.6$ and $X_3=251.8$

Fit

i. Modified exponential curve. (10 Marks)

ii. Logistic curve (10 Marks)

Hence obtain values for $t=5,18,25$

QUESTION FIVE (20 MARKS)

a) Suppose e_t is error term such that $E(e_t)$, $\text{var}(e_t)=\sigma^2$ and $\text{cov}(e_t e_{t'})=0$ where $t \neq t'$.

Using difference of filter, compute the correlation between the error terms. (8 Marks)

b) Consider an AR(2) process given by

$$X_t = X_{t-1} - \tfrac{1}{2} X_{t-2} + e_t$$

Is this process stationary? If so find its ACF. (12 Marks)