



**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE
UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE
ACTUARIAL
4th YEAR 1st SEMESTER 2018/2019 ACADEMIC YEAR
MAIN REGULAR**

COURSE CODE: SAS 401

COURSE TITLE: FURTHER DISTRIBUTION THEORY

EXAM VENUE: STREAM: (Bsc. Actuarial Science)

DATE: EXAM SESSION: SEP-DEC 2018

TIME: 2.00 HOURS

Instructions:

- (i) Answer questions one and any other two.**
- (ii) Candidates are advised not to write on the question paper.**
- (iii) Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question One (20 mks)

a) A car dealer knows from experience that out of 20 vehicles sold, 15% will have insignificant defects, 60% will be fairly damaged and 25% will have severe defects. He wants to determine the probability that out of 20 Vehicles

- i) A maximum of 8 will be severely damaged (Hint: $p=0.25$) (4 mks)
- ii) At least 12 will have moderate defects (Hint: $p=0.6$) (4 mks)

b) Consider the data in the table below

	Number of group Members in a Committee		
Gender	Nominated	Non nominated	
Male	x	m-x	m
Female	r-x	n-(r-x)	n
Total	r	m+n-r	m+n

- i) Obtain the probability of including a given number of males in the committee $p(X=x)$ (3 mks)
- ii) Obtain $E(X)$ (3 mks)
- iii) Obtain $Var(X)$ (3 mks)

c) Suppose

$$f(x, y) = \begin{cases} (x + y); & 0 \leq x \leq 1; 0 \leq y \leq 1 \\ 0; & \text{Otherwise} \end{cases}$$

Obtain

- i) $E(XY)$ (3 mks)
- ii) $Cov(X, Y)$ (3 mks)
- iii) $Cor(X, Y)$ (4 mks)

Assuming X and Y are conditionally independent

d) Let X have a density function

$$f(x) = \begin{cases} e^{-x} & ; & x > 0 \\ 0 & ; & \text{Otherwise} \end{cases}$$

Find the new density function of a random variable $Y=X^2$ (6 mks)

Question Two (20 mks)

Consider

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}(x - \mu)^2\right\}; \quad -\infty < x < \infty; \quad -\infty < \mu < \infty$$

Use moment generating technique to obtain

- i) $E(X)$ (10 mks)
- ii) $\text{Var}(X)$ (10 mks)

Question Three (20 mks)

Suppose that a joint pdf of two random variables X and Y is as follows

$$f(x, y) = \begin{cases} c(x^2 + y); & \text{for } 0 < y \leq (1 - x^2) \\ 0; & \text{Otherwise} \end{cases}$$

Determine

- a) Value of the constant c (4 mks)
- b) $\text{Cov}(X, Y)$ (4 mks)
- c) Pearson $\text{cor}(X, Y)$ (4 mks)
- d) Regression equation between X and Y (4 mks)
- e) Obtain coefficient of determination and interpret the fit statistic (4 mks)

Question Four (20 mks)

Describe the regression between x and y from joint distribution function given by

$$f(x, y) = \begin{cases} 2xy; & 0 < y < x; 0 < x < 2 \\ 0; & \text{Otherwise} \end{cases}$$

Obtain

- i) Cov(X, Y) (4 mks)
- ii) The correlation coefficient (4 mks)
- iii) The regression equation between X and Y (4 mks)
- iv) Interpret the regression parameters (4 mks)
- v) Sketch a scatter diagram between X and Y including line of best fit (4 mks)

Question Five (20 mks)

Let X_1, X_2, \dots, X_n be a random variables such that $X_i \sim x^2(r_i); i = 1, 2, \dots, n$ Let each X_i and X_j be independent. Obtain joint pdf of X_{i1} , and X_2 , hence of

$$f = \frac{x_1/r_1}{x_2/r_2} \quad \{0 < x_i < \infty \quad \forall \quad i = 1, \dots, n\}$$