JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

4TH YEAR 1ST SEMESTER 2018/2019 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SMA 403

COURSE TITLE: TOPOLOGY

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 marks)

a) Let $A = \{1,2,3\}$, determine the power set of A. (3mks)

b) Define the following terms:
   i) A limit point.
   ii) A closed set.
   iii) A topological space. (7mks)

c) Given a set $X = \{x, y, z\}$ and $\tau = \{\emptyset, X, \{x\}, \{z\}, \{x, z\}\}$. Is τ a topology on X? (4mks)

d) Let (X, τ) be a topological space. Then prove that a subset $S \subseteq X$ is closed if and only if it contains all its limit points. (6mks)

e) Let $X = \mathbb{R}$. Define a metric $d = \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ by $d(x, y) = |x - y|$. Show that $d(x, y)$ is a metric space. (6mks)

f) Let $X = \{x, y, z\}$, $\tau_1 = \{X, \emptyset, \{x\}, \{z\}, \{y, z\}\}$ and $\tau_2 = \{X, \emptyset, \{z\}, \{y, z\}\}$. Determine
   i) the coarseness of the topologies. (2mks)
   ii) $\tau_1 \cup \tau_2$. (1mk)
   iii) $\tau_1 \cap \tau_2$. (1mk)

QUESTION TWO (20 marks)

a) If $f: X \to Y$ and $g: Y \to Z$ are homeomorphisms, then show that the composition $g \circ f: X \to Z$ is also a homeomorphism. (5mks)

b) Let the function $f: \mathbb{R} \to \mathbb{R}$ be given by $f(x) = 3x + 1$. Show that f is a homeomorphism. (5mks)

c) Given the set $X = \{a, b, c, d, e\}$ and a collection of some of its subsets $\tau = \{X, \emptyset, \{d\}, \{b, c\}, \{b, c, e\}, \{a, b, c, d\}\}$. Let $A = \{a, c, d\}$.
   Find
   i) The derived set of A. (5mks)
   ii) the interior points of A. (3mks)
   iii) the exterior points of A. (2mks)

QUESTION THREE (20 marks)

a) Define the following terms

i) A $T1$ topological space. (2mks)

ii) Hausdorff space. (2mks)

b) Let (X, τ) be a metrizable topological space. Show that (X, τ) is Hausdorff. (6mks)

c) Show that a topological space (X, τ) is $T1$ if and only if points in X are closed sets. (10mks)

QUESTION FOUR (20 marks)

a) Define the following terms:

i) relative topology. (2mks)

iii) a continuous function. (2mks)

b) Consider the following topologies on $X = \{1,2,3,4,5\}$
τ =$\{X, \phi, \{1\},\{1,2\},\{1,3,4\},\{1,2,3,4\},\{1,2,5\}\}$. List the members of the relative topology τ_A on $A = \{1,2,4\}$. (8mks)

c) Consider the sets $X = \{w, x, y, z\}$ and $Y = \{1,3,4,6\}$. Let
$\tau_x = \{\emptyset, X, \{w\}, \{z\}, \{y, z\}, \{w, x, z\}\}$, $\tau_y = \{\emptyset, Y, \{4\}, \{6\}, \{3,6\}\{1,4,6\}\}$. Let
$g: X \to Y$ be defined as $g(w) = 4$, $g(x) = 1$, $g(y) = 3$ and $g(z) = 6$, determine whether g is a continuous function or not. (8mks)

QUESTION FIVE (20 marks)

a) Let (X, τ) be a topological space. Then show the following:

i) $\emptyset$ and X are closed.

ii) Arbitrary intersections of closed sets are closed.

iii) Finite unions of closed sets are closed. (12mks)

b) Consider the set $A = \{1,2,3,4,5\}$ and $\mathfrak{B} = \{\{2\}, \{3,5\}, \{1,4\}, \emptyset\}$. Determine

i) $\mathfrak{B}$ is a basis for the topology on X (5mks)

ii) The topology τ on X generated by $\mathfrak{B}$ (3mks)