JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE

IN APPLIED STATISTICS

1ST YEAR 1ST SEMESTER 2018/2019 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SAS 803

COURSE TITLE: ESTIMATION THEORY

EXAM VENUE: **STREAM:**

DATE: **EXAM SESSION:**

TIME: 3.00 HOURS

Instructions:

1. **Answer ANY 3 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (20 MARKS)

a) Explain clearly what is meant by weak consistency. (2marks)

b) Let $X_1, X_2, \dots, X_n$ be iid random variables from the uniform $u(a,b)$distribution. Suppose $T_n = 2\bar{x} - a$. Show that T_n is weakly consistent for b when a is known (10marks)

c) Let $X_1, X_2, \dots, X_n$ be iid random variables from a population having p.d.f $f(x) = \frac{1}{\theta^n \Gamma(n)} e^{-x/\theta} x^{n-1} \;:\; x > 0, \theta > 0$. Show that $\frac{\Sigma x^2}{n^2(n+1)}$ is unbiased for θ^2 (8marks)

QUESTION TWO (20 MARKS)

a) Distinguish between method of moment and Maximum Likelihood method of estimation (5marks)

b) Let $X_1, X_2, \dots, X_n$ be iid random variables from a population having p.d.f

$f(x) = \frac{1}{\theta^m \Gamma(m)} e^{-x/\theta} x^{m-1} \;:\; x > 0, m > 0$.

Assuming that

i. m is known

ii. $m = 2$, find MLE of θ (10 marks)

c) Find moment's estimator of θ given that;

$f(x) = \theta^x (1-\theta)^{1-x}\,; x = 0,1;\; \theta \epsilon (0,1)$ (5 marks)

QUESTION THREE (20 MARKS)

a) Distinguish sufficiency from completeness as used in parameter estimation theory. (4 marks)

b) Let $X_1, X_2, \dots, X_n$ are iid random variables from a population with p.d.f

$f(x) = \frac{1}{\beta(\theta,2)} x^{\theta-1}(1-x) \;:\; 0 < x < 1, \theta > 0$. By the factorization criterion, find a sufficient statistic for θ. (8marks)

c) Let $X_1, X_2, \dots, X_n$ are iid $N(\mu, \sigma^2)$. Find a complete sufficient statistic for the population mean and variance. (8 marks)

QUESTION FOUR (20MARKS)

a) Let $X_1, X_2, \dots, X_n$ be iid $N(\mu, \sigma^2)$ random variables where both mean and variance are unknown. Find the UMVUE of

i. μ
ii. σ^2
iii. $d(\mu) = \mu^2$ (10marks)

b) Let $X_1, X_2, \dots, X_n$ be iid Poisson random variables with $f(x_i, \theta) = \frac{e^{-\theta}\theta^{x_i}}{x_i!}$ $x = 0,12, \dots$
Find a UMVUE of θ^2. Does there exist a MVBUE of θ^2. (10marks)

QUESTION FIVE (20 MARKS)

a) Find the estimator of α and β in

$$, \quad f(x) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma\alpha\Gamma\beta} x^{\alpha-1}(1-x)^{\beta-1}, & 0 < x < 1, \alpha > 0, \beta > 0 \\ 0, & otherwise \end{cases}$$

By method of moment. (12 marks)

b) The following data represents the number of earning members (X) and the total monthly income (Y) hundreds of thousands of 15 randomly selected families in an area.

1	1	1	1	1	1	1	2	2	2	3	3	3	4	2
20	15.5	12	8.6	6	8	6	33.5	40.6	50	65.6	20	38	100	35

Assuming a linear relationship of Y and X, estimate the parameters by method of least squares. (8marks)