JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

2ND YEAR 2ND SEMESTER 2018/2019 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SAC 206

COURSE TITLE: ACTUARIAL MATHEMATICS I

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

(a) Define the following terms as used in actuarial mathematics;

i. Contingent probability.

ii. ${}_tq_{xy}$

iii. ${}_tp_{\overline{xy}}$ [5 marks]

(b) Calculate the present value as at 1 January 2003 of an annuity payable annually in arrear for 15 years. The first payment is 500 and subsequent payments increase by 3% per annum compound. [5 marks]

(c) Consider the following function for a newborn

$$S_0(x) = \frac{1}{c}(110 - x)^{\frac{2}{3}},\ 0 \leq x \leq 110$$

i. Calculate c so that this survival function is legitimate and give the limiting age for this model. [2 marks]

ii. Calculate the probability that a newborn will reach to age 65 but die within 20 years following that. [2 marks]

iii. Calculate the expected future lifetime of a newborn. [2 marks]

(d) Given the following ${}_5p_{50} = 0.9$, ${}_{10}p_{50} = 0.8$ and $q_{55} = 0.03$. Find the probability that (56) will die within four years. [5 marks]

(e) Given that $q_{70} = 0.01043$ and $q_{71} = 0.01167$. Calculate

i. ${}_{0.7}q_{70.6}$ assuming a constant force of mortality, [3 marks]

ii. ${}_{0.7}q_{70.6}$ assuming a uniform distribution of deaths. [3 marks]

(f) If $\mu_x = 0.01908 + 0.001(x - 70)$, for $x \geq 55$, calculate ${}_5q_{60}$. [3 marks]

QUESTION TWO

(a) Let X be an age at death random variable. If mortality is described by

$$s(x) = (1 - \frac{x^2}{8100}) \qquad \text{for } 0 \leq x \leq 90$$

Determine

i. $\mathring{e}_0$ and interpret this value.(show your working) [5 marks]

ii. The probability that a life age 35 dies before age 55 years. [2 marks]

iii. $\mu(40)$ [5 marks]

(b) The following is an extract from a standard mortality table.

x	40	41	42
q_x	0.00278	0.00298	0.0032

A substandard table is obtained from this standard table by adding a constant $c = 0.1$ to the force of mortality which results to rates denoted by q_x^s. Calculate the probability that a substandard life (40) will die between ages 41 and 42. [5 marks]

(c) A man makes payments into an investment account of Ksh 200 at time 5, Ksh 190 at time 6, Ksh 180 at time 7, and so on until a payment of Ksh 100 at time 15. Assuming an annual effective rate of interest of 3.5%, calculate: the present value of the payments at time 4. [5 marks]

QUESTION THREE

(a) The following is an extract from a select and ultimate table. Use it to answer the following questions;

[x]	$l_{[x]}$	$l_{[x+1]}$	$l_{[x+2]}$	x+2
40	33519	33485	33440	42
41	33467	33428	33378	43
42	33407	33365	33309	44
43	33340	33294	33231	45
44	33265	33213	33143	46

i. What is the select period? [2 marks]

ii. Calculate the following probabilities; ${}_2p_{[42]}$ and ${}_3q_{[41]+1}$. [4 marks]

iii. Assuming a UDD between integer ages, calculate ${}_{0.5}p_{44}$. [3 marks]

(b) It is given that ${}_{k|}q_0 = 0.1(k+1)$, for $k = 0, 1, 2, 3$. Suppose linear assumption holds between integral ages, find ${}_{2.75}p_0$. [4 marks]

(c) Show that ${}_tp_x = 1 - t \cdot q_x$ under uniform distribution of deaths assumption. [3 marks]

(d) Let X be the age at death random variable. Assume that $X \sim$ DeMoivre's law with omega as 100. Calculate the μ_{30}. [4 marks]

QUESTION FOUR

(a) Consider the following survival function

$$s(x) = 1 - \frac{x}{95},\ 0 \leq x \leq 95$$

i. Derive the expression for the force of mortality for (x), [5 marks]

ii. Derive the expression for ${}_tp_{75}$, [3 marks]

iii. Calculate $E[K_{75}]$. [3 marks]

(b) Suppose that for an initial investment of 1000 dollars you obtain a payment of 400 dollars after one year and 770 dollars after two years. Obtain the yield of this deal. [5 marks]

(c) Calculate the value of ${}_{1.75}p_{45.5}$ on the basis of AM92 mortality table and assuming that deaths are uniformly distributed between integral ages. [4 marks]

QUESTION FIVE

(a) A perpetuity immediate has annual payments. The first payment is 1 and each subsequent payment increases by 1 until the payment reaches 20. The payments stay level thereafter. Find the present value of the perpetuity at an annual effective interest rate of 6%. [5 marks]

(b) The mortality of a certain population is governed by the life table function $l_x = 100 - x$, $0 \leq x \leq 100$. Calculate the values of the following expressions:

i. μ_{30} [3 marks]

ii. $P(T_{30} < 20)$ [2 marks]

iii. $\mathring{e}_{30}$. [3 marks]

(c) The complete life expectation of a life age x, is $\mathring{e}_x$, show that $\mathring{e}_x = \int_0^\infty {}_tp_x dt$. [3 marks]

(d) The survival function of (x) is given by

$$s(x) = (1 - \frac{x}{\omega})^{2.5}, \; 0 \leq x \leq \omega$$

If $\mu_{80} = 0.05$, calculate and interpret $\mathring{e}_{60:\overline{25}|}$. [5 marks]