JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

4TH YEAR 2ND SEMESTER 2018/2019 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SAC 404

COURSE TITLE: COMPUTATIONAL FINANCE

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION 1 [COMPULSORY] [30 Marks]

(a) Define the following terms is respect of securities trading:

(i)Intrinsic value [**2 Marks**]

(ii)Hedging [**2 Marks**]

(iii)Short position [**2 Marks**]

(b)Consider a three-period binomial tree model for a stock price process S_t , under which the stock price either rises by 18% or falls by 15% each month. No dividends are payable. The continuously compounded risk-free rate is 0.25% per month. Let $S_0 = 85$. Consider a European put option on this stock, with maturity in three months (i.e. at time t = 3) and strike price Kshs.90.

(i) Calculate the price of this put option at time $t = 0$. [**6 Marks**]

(ii) Calculate the risk-neutral probability that the put option expires out-of-the money. [**2 Marks**]

(c) Consider a particular stock and denote its price at any time t by S_t . This stock pays a dividend D at time $T^{'}$.

Let C_t and P_t be the price at time t of a European call option and European put option respectively, written on S, with strike price K and maturity $T \geq T' \geq t$. The instantaneous risk-free rate is denoted by r.

Prove the put-call parity in this context by adapting the proof of standard put-call parity. [**6 Marks**]

(d)Let p_t denote the value at time t (measured in years) of a European put option on a non-dividend-paying stock with price S_t . The option matures at time T and has a strike price K. The continuously compounded risk-free rate of interest is r. Derive a lower bound for p_t in terms of S_t and K.

[6 Marks]

(e)A binomial lattice is used to model the price of a non-dividend-paying share up to time T . The interval $(0, T)$ is subdivided into a large number of intervals of length $\delta t = \frac{T}{n}$. It is assumed that, at each node in the lattice, the share price is equally likely to increase by a factor u or decrease by a factor d , where $u = e^{\mu+\sigma\sqrt{\delta t}}$ and $d = e^{\mu-\sigma\sqrt{\delta t}}$. The movements at each step are assumed to be independent.

Show that, if the share price makes a total of X_n up jumps, the share price at time T will be:

$$S_T = S_0 exp\left\{+\sigma\sqrt{T}\left(\frac{2X_n - n}{\sqrt{n}}\right)\right\}$$

where S_0 denotes the initial share price. **[8Marks]**

QUESTION 2 [20 Marks]

Consider a European call option with price ct written on an underlying non-dividend paying security with price S_t at current time t.

(a) State whether each of the following changes in underlying factors would increase or reduce the price of this option:

(i) a fall in the price of the underlying security

(ii) an increase in the strike price of the option

(iii) an increase in the volatility of the underlying security price

(iv) a fall in the risk-free rate of interest

You should assume that each change occurs on a standalone basis, i.e. all other factors are unchanged.

[**4 Marks**]

(b) Explain each of your statements in part (a). [**8 Marks**]

(c)Consider a European put option with price p_t written on the same underlying security, with the same strike price K and the same maturity T as the call option described above.

The continuously compounded risk-free rate of interest is r.

(i) Write down a formula that relates the values of c_t and p_t.[**4 Marks**]

The call option has value Kshs.0.50 at time $t = 0$, and the put option has value Kshs.1.00. Both options are written on a security with current value $S_0 = 5$, and both options have strike price Kshs.6.00 and maturity $T = 3$ years.

(ii) Determine the continuously compounded risk-free rate r.[**4 Marks**]

QUESTION 3 [20 Marks]

The current price of a non-dividend paying stock is Kshs.65 and its volatility is 25% per annum. The continuously compounded risk-free interest rate is 2% per annum. Consider a European call option on this share with strike price Kshs.55 and expiry date in six months time. Assume that the Black-Scholes model applies.

(a)Calculate the price of the call option . [**10 Marks**]

(b)Define algebraically the delta of the call option. [**2 Marks**]

(c) Calculate the value of the delta of the call option. [**4 Marks**]

(d)Calculate the value of the delta of a European put option written on the same underlying, with the same strike and maturity as above.

[**4 Marks**]

QUESTION 4 [20 Marks]

Consider a call option on a non-dividend paying stock S when the stock price is Kshs.15, the exercise price, K, is Kshs.12, the continuously compounded risk-free rate of interest is 2% per annum, the volatility is 20% per annum and the time to maturity is three months. (a) Calculate the price of the option using the Black-Scholes model. [**10 Marks**]

(b) Determine the (risk neutral) probability of the option expiring in the money. **[2 Marks]**

A special option called a digital cash-or-nothing option has a payoff in three months time of:

$$\begin{cases} 1 & S_T < K \\ 0 & S_T > K \end{cases}$$

(c) Calculate the price of the digital option. **[4 Marks]**

(d) Describe the limitations of the Black-Scholes model. **[4 Marks]**

QUESTION 5 [20 Marks]

Explain the similarities and differences in the following three interest rate models:

- the Hull White model
- the Cox-Ingersoll-Ross model
- the Vasicek model

[20 Marks]