JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

2018/2019

YEAR ONE SEMESTER TWO ACADEMIC YEAR

MAIN

COURSE CODE: SAS 102

COURSE TITLE: PROBABILITY AND DISTRIBUTION THEORY I

DATE: 26/4/19 **EXAM SESSION: 9.00 – 11.00 AM**

TIME: 2.00 HOURS

Instructions:

1. **Answer Question ONE and ANY other two questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY) – (30 MARKS)

a) The weekly amount of time X (in hours) that a manufacturing plant is down (due to maintenance or repairs) has an exponential distribution with mean 8.5 hours.

The cost of the downtime, due to lost production and maintenance and repair costs, is modeled by $Y = 15 + 5X + 1.2X^2$ in millions of shillings.

Determine the Expected cost of the downtime. (6 Marks)

b) The joint probability function for the random variables X and Y is tabulated as shown

	Y=6	Y=8	Y=10
X=1	0.2	0.2	0.2
X=2	0	0	0
X=3	0.2	0.2	0.2

Determine:

i. $E(X)$
ii. $E(Y)$
iii. $E(XY)$ (6 Marks)

c) The random variables X and Y have joint probability density function given by

$$f(x,y) = \begin{cases} 45xy^2(1-x)(1-y^2), & 0 < x < 1, 0 < y < 1 \\ 0, & otherwise \end{cases}$$

Determine marginal density functions $f(x)$ and $f(y)$ (6 Marks)

d) Let $f(x) = \begin{cases} 5x^4, & 0 < x < 1 \\ 0, & otherwise \end{cases}$. Determine the $p.d.f$ of $Y = X^3$ (5 Marks)

e) Hotel customers are charged for the amount of salad they take. Sampling suggests that the amount of salad taken is uniformly distributed between 5 grams and 15 grams. Let X represent the salad weight in a plate, find

i. the expected Value and the Variance of X.
ii. the probability that a customer will take between 12 and 15 grams of salad? (7 Marks)

QUESTION TWO (20 MARKS)

a) The life of a roller bearing X follows a Weibull distribution with parameters β and σ. The distribution of X may be presented as follows:

$$f(x) = \begin{cases} \frac{\beta}{\sigma}\left(\frac{x}{\sigma}\right)^{\beta-1} exp^{-\left(\frac{x}{\sigma}\right)^{\beta}}, & x > 0, \beta > 0, \sigma > 0 \\ 0, & 0therwise \end{cases}$$

Determine

i. The mean time until failure of a bearing.
ii. At $\beta = 2$ and $\sigma = 10000\ hours$ the probability that a bearing lasts at least 8000 hours.
iii. If 10 bearings are in use and failures occur independently and $= 2$, $\sigma = 10000\ hours$, the probability that all the ten bearings will last at least 8,000 hours. (14 marks)

b) Consider a bivariate function.

$$f(x,y) = \begin{cases} k(6-x-y), & 0<x<2, 2<y<4 \\ 0, & otherwise \end{cases}$$

Obtain $P(X+Y<4)$ (6 Marks)

QUESTION THREE (20 MARKS)

a) The gamma distribution $f(x) = \begin{cases} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^{\alpha}\Gamma(\alpha)}, & x>0, \beta>0, \alpha>0 \\ 0, & x \le 0 \end{cases}$, with scale parameter β and shape parameter α reduces to a Chi-square distribution with v degrees of freedom when $\beta = 2$ and $E(X) = v$. Based on this information, obtain an expression for the Chi- square density function hence derive an expression for $E(X^3)$ (12 Marks)

b) Consider a two dimensional random variables (X,Y) having a joint probability distribution function;

$$f(x,y) = \begin{cases} 2xy, & 0 \le x \le k, \quad 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$

Where k is constant. Find

i) The value of k . (4 Marks)

ii) The marginal probability distribution function of Y. (4 Marks)

QUESTION FOUR (20 MARKS)

a) The joint probability function of two discrete random variables X and Y is given by $f(x,y) = \begin{cases} k(2x+y), & 0 \le x \le 2, 0 \le y \le 3 \\ 0, & otherwise \end{cases}$

Obtain

i. the value of k hence $P(X \ge 1, Y \le 2)$. (5 Marks)

ii. $Cov\ (X,Y)$ (8 Marks)

b) Given $f(x,y) = \begin{cases} \frac{1}{9}(xy), & 0<x<2, 0<y<3 \\ 0, & otherwise \end{cases}$ obtain $var\ (Y/X=x)$ (7 Marks)

QUESTION FIVE (20 MARKS)

a) Given $(x,y) = \begin{cases} \frac{1}{8}(x+y), & 0<x<2, 0<y<2 \\ 0, & otherwise \end{cases}$.

Determine the joint probability distribution function of the new random variables: $U = x+y$ and $V = 2x-y$ (12 Marks)

b) The joint probability mass function of two random variables X and Y is defined as follows

$$f(x,y) = \begin{cases} c(kx+y), & 0<x<2, 1<y<3 \\ 0, & otherwise \end{cases}$$

Where c and k are constants. Given that $(E(Y) = 2$. Find c and k (8 Marks)