**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**DRAFT - EXAMINATIONS 2012/2013**

**SEMESTER 1 FIRST YEAR MSc EXAMS**

**COURSE CODE: SMA 840**
**COURSE TITLE: METHODS**

**DATE : Aug, 2013** **TIME: 3hrs**

**INSTRUCTIONS**

ATTEMPT ANY **THREE** QUESTIONS

**Show all the necessary working**

**Question1[20 marks]**

Radio telegraphic transformer $L-R-C$ electrical network in figure1 below, shows the primary current $I_1(t)$ and secondary current $I_2(t)$ which satisfies the Kirchhoff's laws

$$L_1 I_1' + M I_2' + \frac{1}{C_1}\int_0^t I_1 dx = E(t) \quad .$$

$$M I_1' + L_2 I_2' + R I_2 + \frac{1}{C_2}\int_0^t I_2 dx = 0$$

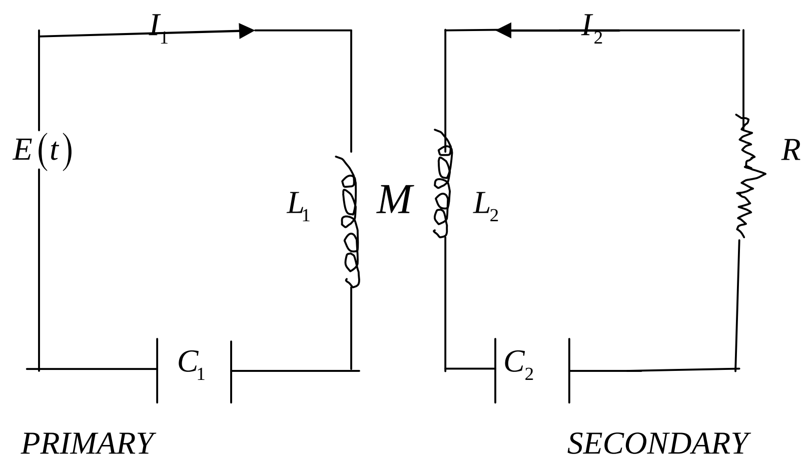


Fig. 1

Examine the oscillations generated by the secondary current $I_2(t)$ of the system, if initially there is no current or charge. Take the forcing function as $E(t) = ¢(t)$

**Question2 [20 marks]**

(a). Compute the Laplace transform of (i) $te^{-40t}\sin 3t$ (ii) $\left[\dfrac{\cos 3t - \cos 2t}{t}\right]$ [8 marks]

(b) If $F(s) = \dfrac{4s+3}{s^3+2s^2+s+2} = \dfrac{4s+3}{(s^2+1)(s+2)}$ use the Laplace transforms and convolution theorem to find $f(t)$. [8 marks]

(c) .Given the full wave rectifier function $f(t) = \begin{cases} E\sin 200t, & 0 < t < ª/200 \\ 0, & ª/200 < t < ª/100 \end{cases}$

(i) State the period of the periodic rectifier function above.

(ii) Find the Laplace transform of the full wave rectifier function. [4 marks]

**Question3 [20marks]**

Define a system of first order linear ordinary differential equations by

$$\dot{\underset{\sim}{X}} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -8 & -5 & -3 \end{pmatrix} \underset{\sim}{X} + \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} .$$

(a) Compute $e^{At}$ the exponential matrix

of the system and verify that $\left[ e^{At} \right]_{t=0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b)Show that $e^{At} = \Phi(t)\Phi^{-1}(0)$ where $\Phi(t)$ is the fundamental matrix of the system.

**Question4 [20marks]**

(a) Apply the Laplace transform method to solve the simultaneous equations

$\frac{dx}{dt} - 5y = -3x, \ \frac{dy}{dt} - y = -x: \ x(0) = 2, y(0) = 1$ [13 marks]

(b) Determine the value of $y$ which satisfies the ordinary differential equation

$xy'' + (1-x)y' + my = 0;$ subject to $y(0) = \sim, \ y'(0) =$ , $m$ positive integer.

[7 marks]

**Question5 [20marks]**

(a)Find all the solutions for the boundary-value problem

$y'' + ª y = 0, y(0) = 0 \ , y(2®) = 0 \ ; \ ª > 0$

Show that ª satisfies the relation $ª_k = \frac{k^2}{4}$ [8 marks]

**(b)** Solve the initial boundary value problem

$u_t = u_{xx}, \quad 0 < x < 1, t > 0$ satisfying the conditions

$u(0,t) = 1, \ u(1,t) = 1 \quad 0 < x < 1, t > 0$

$u(x,0) = 1 + \sin ® x, \quad 0 < x < 1$ [12 marks].

**LAPLACE TRANSFORMS TABLE**

| $f(t)$ | Laplace transform of $f(t)$ |
|---|---|
| $1$ | $\frac{1}{s}$ |
| $t$ | $\frac{1}{s^2}$ |
| $e^{at}$ | $\frac{1}{s-a}$ |
| $\cos bt$ | $\frac{s}{s^2+b^2}$ |
| $\sin bt$ | $\frac{b}{s^2+b^2}$ |
| $e^{-at}\sin bt$ | $\frac{b}{(s+a)^2+b^2}$ |
| $e^{-at}co\,s\,bt$ | $\frac{(s+a)}{(s+a)^2+b^2}$ |
| $e^{-at}t^n$ | $\frac{\Gamma(n+1)}{(s+a)^{n+1}} \quad n>-1$ |
| $t^n$ | $\frac{n!}{s^{n+1}}$ |
| $e^{-at}t^n$ | $\frac{n!}{(s+a)^{n+1}}$ |
| $\frac{dy}{dt}$ | $sY-y_0 \quad ; \quad Y=L(y)$ |
| $\frac{d^2y}{dt^2}$ | $s^2Y-sy_0-y_0' \;;\; Y=L(y)$ |
| $\frac{d^ny}{dt^n}$ | $s^nY-s^{n-1}y_0-s^{n-2}y_0-s^{n-3}y_0-....$ $-sy^{(n-2)}(0)-y^{(n-1)}(0)$ |
| $u_t$ | $sU(x,s)-u(x,0); \quad U(x,s)=L\left[u(x,t)\right]$ |
| $u_{tt}$ | $s^2U(x,s)-su(x,0)-u_t(x,0);$ |
| $u_{x^m}$ | $\frac{d^m}{dx^m}(U(x,s))$ |
| $u_{xt}$ | $s\frac{d}{dx}U(x,s)-\frac{d}{dx}u(x,0)$ |
| $J_0(t)$ | $\frac{1}{\sqrt{s^2+1}}$ |

| | |
|---|---|
| $t^n f(t)$<br><br>$\dfrac{f(t)}{t}$ | $(-1)^n \dfrac{d^n \{F(s)\}}{ds^n}$<br><br>$\int_s^\infty F(s)\,ds$ |
| $f^{(n)}(t)$ | $s^n F(s) - s^{n-1} f(0) - s^{n-2} f(0) - ....sf^{(n-2)}(0) - f^{(n-1)}(0)$ |
| $J_0(t)$ | $\dfrac{1}{\sqrt{1+s^2}}$ |
| | |
| $\dfrac{\partial u(x,t)}{\partial t}$ | $sU(x,s) - u(x,0)$ |
| $\dfrac{\partial^2 u(x,t)}{\partial t^2}$<br>$e^{At}$ | $s^2 U(x,s) - su(x,0) - u_t(x,0)$<br><br>$(sI - A)^{-1}$ |

**LAPLACE TRANSFORMS**

| | |
|---|---|
| $\dfrac{\partial u(x,t)}{\partial t}$ | $sU(x,s) - u(x,0)$ |
| $\dfrac{\partial^2 u(x,t)}{\partial t^2}$ | $s^2 U(x,s) - su(x,0) - u_t(x,0)$ |
| $\dfrac{\partial u(x,t)}{\partial x}$ | $\dfrac{dU(x,s)}{dx}$ |
| $\dfrac{\partial^2 u(x,t)}{\partial x^2}$ | $\dfrac{d^2 U(x,s)}{dx^2}$ |
| $\dfrac{\partial^2 u(x,t)}{\partial t \partial x}$ | $s\dfrac{dU(x,s)}{dx} - \dfrac{du(x,0)}{dx}$ |
| $L^{-1}[F(s)G(s)] = \int_0^t f(t-u)g(u)du$ | |
| $H(t-a)$ | $\dfrac{e^{-as}}{s}$ |
| $\cosh at$ | $\dfrac{s}{s^2 - a^2}$ |
| $\sinh at$ | $\dfrac{a}{s^2 - a^2}$ |