JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF HEALTH SCIENCES

UNIVERSITY EXAMINATION FOR BED AND ACT SCIENCE

2nd YEAR 2ND SEMESTER 2018/2019 ACADEMIC YEAR

MAIN CAMPUS – INSTITUTIONAL BASED

COURSE CODE: SMA201

COURSE TITLE: LINEAR ALGEBRA II

EXAM VENUE: **STREAM: BED AND ACT SCIENCE Y2S2**

DATE: 19/08/19

TIME: 2 HOURS **EXAM SESSION: 9.00 – 11.00am**

Instructions:

1. **Answer question1 and any other 2 questions.**
2. **Candidates are advised not to write on the question paper**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

Question1 [30marks] Compulsory

(a) (i)Given matrix $M = \begin{bmatrix} 11 & 0 & 12 \\ 6 & 24 & 0 \\ 1 & 0 & 10 \end{bmatrix}$

Compute M^t the transpose of M and determinant of M . [5marks]

(ii)If $A = \begin{bmatrix} 11-4i & i & 12 \\ 6+2i & 24+i & 0 \\ 1 & i & i \end{bmatrix}$

Find the matrix A^* the ***adjoint*** of **A**. [5marks]

(b) Suppose the mapping $L : R^2 \to R^2$ with $L\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x-y \\ x+y \end{bmatrix}$

(i) Show that L is linear. [6marks]

(ii)Determine A the matrix of L with respect to the ordered basis $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$ [7 marks]

(c) Let the binary rules $\oplus , \otimes$ be defined on the R^2 vector space by : $\begin{pmatrix} x \\ y \end{pmatrix} \oplus \begin{pmatrix} u \\ v \end{pmatrix} = xu + yv$

; $\begin{pmatrix} x \\ y \end{pmatrix} \otimes \begin{pmatrix} u \\ v \end{pmatrix} = xu - yv - 5$

$(i)Compute$ $\begin{pmatrix} 0 \\ 0 \end{pmatrix} \oplus \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} x \\ y \end{pmatrix} \oplus \begin{pmatrix} x \\ y \end{pmatrix}$, $\begin{pmatrix} x \\ y \end{pmatrix} \otimes \begin{pmatrix} x \\ y \end{pmatrix}$: $\begin{pmatrix} x \\ y \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ **[7marks]**

(ii)State giving reasons which of the rules $\oplus , \otimes$ is not an inner product on the R^2 vector space[7marks]

Question2 [20marks]

(a)(i) Without using direct computation , show that $-3,\ -3\ ,\ 1$ eigenvalues of the matrix $A=\begin{pmatrix}1 & -4 & -4\\ 8 & -11 & -8\\ -8 & 8 & 5\end{pmatrix}$. [4marks]

(ii) Verify that $\left[A+3I\right]^2\left[A-I\right]=\underline{0}_{3\times3}$ [5marks]

(b) (i) For what values of the constants a,b,d does the matrix equation $a\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -4\\ 0 & 4 & 0\end{bmatrix}+b\begin{bmatrix}0 & 0 & 4\\ 0 & 0 & 0\\ -4 & 0 & 0\end{bmatrix}+d\begin{bmatrix}0 & -4 & 0\\ 4 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$ hold ? [4marks]

(ii) Determine the specific values of the constants a,b,d such that the set of 3by3 matrices $\left\{\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -4\\ 0 & 4 & 0\end{bmatrix},\begin{bmatrix}0 & 0 & 4\\ 0 & 0 & 0\\ -4 & 0 & 0\end{bmatrix},\begin{bmatrix}0 & -4 & 0\\ 4 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\right\}$ is linearly independent

[7marks]

Question3 [20marks]

Given $A=\begin{pmatrix}8 & -2 & -3 & 1\\ 7 & -1 & -3 & 1\\ 6 & -2 & -1 & 1\\ 5 & -2 & -3 & 4\end{pmatrix}$ is matrix of linear operator T

(a) If $v_0=\left[0\ ,\ 0,\ 0,\ 0\right]^t$ $v_1=\left[10\ ,\ 10,\ 10,\ 10\right]^t$, $v_2=\left[7\ ,\ 7,\ 7,\ 0\right]^t$, $v_3=\left[4\ ,\ 10,\ 4,\ 4\right]^t$, $v_4=\left[3,3,5,3\right]^t$,evaluate Av_0 Av_1 , Av_2 , Av_3 , Av_4 . [5 marks]

(b) Find $\lambda_1\ ,\lambda_2\ ,\ \lambda_3\ ,\ \lambda_4$, the eigenvalues of T and set $U=\{u_1\ ,u_2\ ,\ u_3\ ,\ u_4\}$ the corresponding eigenvectors

(c) Prove that the set of eigenvectors is linearly independent [8marks]

(d) Evaluate $\left(\lambda_1+\lambda_2+\lambda_3+\lambda_4\right)-trace\left(A\right)$ [4marks]

Question4 [20 marks]

Consider the vector space of R^4 with the inner product $\langle\ ,\ \rangle$:

$\langle \underline{x}, \underline{y} \rangle = 4x_1y_1 + 4x_2y_2 + x_3y_3 + x_4y_4$; $\underline{x} = [x_1, x_2, x_3, x_4]$, $\underline{y} = [y_1, y_2, y_3, y_4]$, $\underline{0} = [0,0,0,0]$,
$x_i, y_i \in R$; $\underline{x}$, $\underline{y} \in R^4$

(a) Show that $\langle \underline{x}, \underline{x} \rangle > 0$ **[4 marks]**

(b) Show that $\langle \underline{x}, \underline{y} \rangle = \langle \underline{y}, \underline{x} \rangle$ **[4 marks]**

(c) Determine $\langle \underline{x}, \underline{0} \rangle$, $\langle \underline{0}, \underline{y} \rangle$, $\langle \underline{0}, \underline{0} \rangle$ **[4 marks]**

(d) Apply the Gram-Schmidt process to the set of linearly independent vectors

$\{v_1 = [1,1,-1,-1], v_2 = [1,1,1,1], v_3 = [-1,-1,-1,1], v_4 = [1,0,0,1]\}$
to obtain orthogonal set of vectors $\{w_1, w_2, w_3, w_4\}$. **[8 marks]**

Question5 [20marks]

Let B be the matrix of linear operator T on n-dimensional vector space V over F with respect to the standard ordered basis for V.

(a) Explain what is meant by (i) v is an eigenvector of T, $v \in V$ (ii) λ is an eigenvalue of T, $\lambda \in F$
[6 marks]

(b) State the relationship between T and B [2marks]

(c) If matrix $B = \begin{pmatrix} 1 & -9 \\ -1 & 1 \end{pmatrix}$ find

(i) λ_1, λ_2 the eigenvalues of T and v_1, v_2 the corresponding eigenvectors [8marks]

(d) Confirm that B is diagonalizable. [2marks]

(e) Diagonalize matrix B. [2marks]