JARAMOGI OGINGA ODINGA UNIVERSITY OF

SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND STATISTICS

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE WITH IT

3RD YEAR 2ND SEMESTER 2019/2020 ACADEMIC YEAR

MAIN (RESIT)

COURSE CODE: SAC 302

COURSE TITLE: METHODS OF ACTUARIAL INVESTIGATIONS II

EXAM VENUE: **STREAM: (BSc)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

(a) Explain what is meant by the principle of correspondence. [2 marks]

(b) Describe briefly some validity checks that can be carried out to ascertain the reliability of the data to be used for a mortality investigation. [4 marks]

(c) You have been given the information from the mortality experience of a large life office covering the years 2007 and 2008. Your information includes the number of people aged x nearest birthday at 1 January 2007, 2008 and 2009, and the number of deaths during 2007 and 2008 aged x nearest birthday. You have derived crude central rates of mortality μ_x . State the assumptions that you had to make. [3 marks]

(d) A mortality investigation is being conducted between the dates 1 January 2008 and 31 December 2011. The data from four lives under consideration was as follows:

Life	Date of birth	Date of entry	Date of exit	Mode of exit
Pele	11.11.77	24.3.86	29.12.11	Death
Johan	1.9.84	30.8.09	–	Did not leave
Gray	10.2.83	10.10.07	21.6.08	Surrender
Diego	8.2.86	10.8.09	–	Did not leave

i. Assuming that the day of entry, but not the day of exit, counts in the exposed to risk, calculate the number of days of exposure contributed to the central exposed to risk by each life at each age. [12 marks]

ii. State what modifications, if any, you would need to make if you were determining the initial exposed to risk and not the central exposed to risk. [3 marks]

(e) A graduation of a set of mortality rates from age 25 to age 64 has 15 positive individual standardised deviations, which occurred in 8 groups.

Carry out two tests to check the suitability of this graduation. [6 marks]

QUESTION TWO

(a) i. If mortality follows Gompertz Law such that $\mu_x = 0.00003 \times 1.1^x$, calculate the values of μ_{80}, μ_{81}, μ_{82}, μ_{83} (to 7 decimal places) and the first, second and third differences derived from these quantities. [3 marks]

ii. Recalculate differences using the observed rates $\hat{\mu}_{80} = \mu_{80} - 0.01$, $\hat{\mu}_{81} = \mu_{81}$, $\hat{\mu}_{82} = \mu_{82}$, $\hat{\mu}_{83} = \mu_{83}$. [2 marks]

iii. Use the values calculated in (i) and (ii) to explain why the test for smoothness usually requires the third differences of the graduated rates to be small, rather than the differences of a lower or a higher order. [3 marks]

(b) i. For a mortality investigation you have been given data relating to the initial exposed to risk and number of deaths for ages 60 to 94. Describe how you would carry out a graphical graduation and show how you would derive approximate 95% confidence intervals for the age specific mortality rates. [7 marks]

ii. Describe how you would test your graduated rates for smoothness. [1 mark]

(c) In the investigation the following data have been collected.

Age last birthday	Initial exposed to risk	Number of deaths
$60-64$	20500	374
$65-69$	27800	892
$70-74$	30100	1475
$75-79$	26700	2158
$80-84$	17700	2251
$85-89$	9200	1780
$90-94$	2300	630

State, giving reasons, whether the graphical method is appropriate for the graduation of this experience, or whether a different method would be more suitable. [4 marks]

QUESTION THREE

(a) The table below shows the crude and graduated mortality rates for part of the relevant age range, together with the exposed to risk at each age and the standardised deviation at each age.

x	$\mathring{\mu}_{x+\frac{1}{2}}$	$\hat{\mu}_{x+\frac{1}{2}}$	E_x^c
50	0.08127	0.07941	340
51	0.0877	0.08438	320
52	0.09439	0.10345	300
53	0.10133	0.092	290
54	0.10853	0.092	250
55	0.116	0.1	200
56	0.12373	0.11176	170
57	0.13175	0.12222	180

Test this graduation for:

i. overall goodness-of-fit

ii. bias; and

iii. the existence of individual ages at which the graduated rates depart to a substantial degree from the observed rates. [7 marks]

(b) i. Explain why it might be difficult to ensure the principle of correspondence is adhered to, and give a specific example of an investigation where this may be the case. [3 marks]

ii. An actuary was asked to investigate the mortality of lives in a particular geographical area. Data are available of the population of this area, classified by age last birthday, on 1 January in each year. Data on the number of deaths in this area in each calendar year, classified by age nearest birthday at death, are also available. Derive a formula which would allow the actuary to estimate the force of mortality at age $x+f$, μ_{x+f} , in a particular calendar year, in terms of the available data, and derive a value for f. [6 marks]

iii. List four factors other than geographical location which a government statistical office might use to subdivide data for national mortality analysis. [2 marks]

(c) At a particular age there are 922 deaths, compared to 950 expected. Calculate the approximate individual standardised deviation for that age. [2 marks]

QUESTION FOUR

(a) i. Describe three shortcomings of the χ^2 test for comparing crude estimates of mortality with a standard table and why they may occur. [3 marks]

ii. The following table gives an extract of data from a mortality investigation conducted in the rural highlands of a developed country. The raw data have been graduated by reference to a standard mortality table of assured lives.

Age	Expected deaths	Observed deaths	z_x
60	36.15	35	−0.191
61	28.92	24	−0.915
62	31.34	27	−0.775
63	38.01	35	−0.488
64	26.88	32	0.988
65	37.59	36	−0.259
66	33.85	34	0.026
67	26.66	32	1.034
68	22.37	26	0.767
69	18.69	33	3.310
70	18.24	22	0.880

For each of the three shortcomings you described in (i):

A. name a test that would detect that shortcoming.

B. carry out the test on the data above. [12 marks]

iii. Comment on your results from (ii). [3 marks]

(b) List four factors other than age and smoker status by which life insurance mortality statistics are often subdivided. [2 marks]

QUESTION FIVE

(a) An investigation into mortality collects the following data:
θ_x= total number of policies under which death claims are made when the policyholder is aged k last birthday in each calendar year.
$P_x(t)$ = number of in-force policies where the policyholder was aged x nearest birthday on 1 January in year t.

i. Obtain an expression, in terms of the $P_x(t)$, for the central exposed to risk, E_x^c , which corresponds to the claims data and which may be used to estimate the force of mortality in year t at each age x, μ_x . State any assumptions you make. [5 marks]

ii. Comment on the effect on the estimation of the fact that the θ_x relate to claims, rather than deaths, and the $P_x(t)$ relate to policies, not lives. [4 marks]

(b) The actuary to a large pension scheme carried out an investigation of the mortality of the schemes pensioners over the two years from 1 January 2005 to 1 January 2007.

i. List the data required by the actuary for an exact calculation of the central exposed to risk for lives aged x. [2 marks]

ii. The following is an extract from the data collected by the actuary.

Age nearest birthday	*No. of pensioners at* 1.1.2005	1.1.2006	1.1.2007	*Deaths during* 2005	2006
63	1248	1312	1290	10	6
64	1465	1386	1405	13	15
65	1678	1720	1622	16	23
66	1719	1642	1667	22	19
67	1686	1695	1601	19	25

A. Derive an expression that could be used to estimate the central exposed to risk using the available data. State any assumptions you make. [4 marks]

B. Use the data to estimate μ_{64} . State any further assumptions that you make. [2 marks]

(c) A graduation of a set of crude mortality rates is tested for goodness-of-fit using a chi-squared test.

Discuss the factors to be considered in determining the number of degrees of freedom to use for the test statistic. [3 marks]