



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE

ACTUARIAL

SPECIAL RESIT 2020/2021 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SAS 303

COURSE TITLE: Theory of Estimation

EXAM VENUE:

STREAM: (BSc. Actuarial)

DATE:

EXAM SESSION:

TIME: 2.00 HOURS

Instructions:

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (20 MARKS)

- a) Define clearly the following terms as used in estimation theory.
- Consistency
 - Completeness
 - Unbiasedness
 - Sufficiency
- (8marks)
- b) Let X_1, X_2, \dots, X_n be iid Poisson random variables with $f(x_i, \theta) = \frac{e^{-\theta} \theta^{x_i}}{x_i!}$ $x = 0, 1, 2, \dots$. Show that \bar{x} is consistent for θ (8 marks)
- c) Let X_1, X_2, \dots, X_n be iid $N(\mu, \sigma^2)$. Find a joint sufficient statistic for $\theta = (\mu, \sigma^2)$ (8marks)
- d) Let x_1, x_2, \dots, x_n be a random sample of n observations from a population with p.d.f $f(x) = \theta x^{\theta-1}$, $0 < x < 1$. Find the Maximum Likelihood estimator of θ . (6 marks)

QUESTION TWO (20 MARKS)

- a) Let x_1, x_2, \dots, x_n be a random sample of n observations from a population having p.d.f $f(x) = \begin{cases} \frac{2}{\theta^2}(\theta - x), & 0 \leq x \leq \theta \\ 0, & \text{otherwise} \end{cases}$. Obtain the mean of this distribution hence show that $3\bar{x}$ is consistent for θ (10marks)
- b) Let x_1, x_2, \dots, x_n be a random sample of size n from $N(\mu, \sigma^2)$. Find the Cramer Rao lower bound for estimation of the square of the mean. (10marks)

QUESTION THREE (20 MARKS)

Find the estimator of α and β in

$$f(x) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma\alpha\Gamma\beta} x^{\alpha-1}(1-x)^{\beta-1}, & 0 < x < 1, \alpha > 0, \beta > 0 \\ 0, & \text{otherwise} \end{cases}$$

By method of moment.

(20 marks)

QUESTION FOUR (20 MARKS)

- a) Suppose we have a random sample of size $2n$ from a population denoted by X where $(X) = \mu$, $\text{Var}(X) = \sigma^2$. Let $\bar{X}_1 = \frac{1}{2n} \sum_{i=1}^{2n} (X_i)$, $\bar{X}_2 = \frac{1}{n} \sum_{i=1}^n (X_i)$ be two estimators of μ . Investigate \bar{X}_1 and \bar{X}_2 for unbiasedness and consistency. Which between the two

is the better estimator of μ . (12 marks)

- b) Use the Lehmann Scheffe method of construction of minimal sufficient statistics to find the minimal sufficient statistic for $\theta = (\mu, \sigma^2)$ given x_1, x_2, \dots, x_n are iid random variables from $N(\mu, \sigma^2)$ (8 marks)

QUESTION FIVE (20 MARKS)

- a) Let x_1, x_2, \dots, x_n be a random sample from some population with finite mean μ and finite variance σ^2 . Show that the sample variance $S^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$ is biased for the population variance. (10marks)

- b) Let x_1, x_2, \dots, x_n be a random sample of n observations from a population having p.d.f
- $$f(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & , 0 \leq x \leq \infty \\ 0, & \text{otherwise} \end{cases}$$
- Find unbiased estimators of θ and θ^2 (10 marks)