



**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL  
SCIENCES**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN  
ACTUARIAL SCIENCE**

**SPECIAL RESIT 2020/2021 EXAMINATION**

**MAIN REGULAR**

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**COURSE CODE: SAS 408**

**COURSE TITLE: Multivariate Methods**

**EXAM VENUE: STREAM: (BSc Actuarial Science)**

**DATE: EXAM SESSION: JAN-APRIL 2020**

**TIME: 3.00 HOURS**

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**Instructions:**

- 1. Answer questions one and any other two.**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**
- 4. All computations and data analysis to be done with any statistical software.**

### QUESTION ONE (30 Marks)

Use the KEArrests\_Nov2020 (KEArrests) data provided to answer the following questions. The data set contains a hypothetical/simulated statistics, in arrests per 100000 residents for assault, murder, and rape in each of the Kenyan 47 counties in the year 2018. Also given is the percent of the population living in urban areas. Use R to solve the following questions.

- Plot a bivariate boxplot for the KEArrest. Explain the graph. (2 marks)
- Obtain a correlation matrix making sense of the coefficients. (2 marks)
- Compute the variance-covariance matrix (2 marks)
- Evaluate the trace of the var-cov matrix (2 marks)
- Compute the cross product matrix of the var-cov matrix (2 marks)
- Find the determinant and inverse of the var-cov matrix (2 marks)
- Obtain the eigen values and corresponding eigen vectors of the var-cov matrix (2 marks)
- Provide spectral decomposition of the var-cov matrix (2 marks)
- When is a matrix said to be symmetric positive definite? (2 marks)
- Evaluate whether the var-cov matrix is symmetric positive definite (2 marks)
- Convert the var-cov matrix to a correlation matrix, calculate the eigen values and eigen vectors and verify that the eigen vectors are orthogonal (2 marks)
- Assume that X is the KEArrest data. Obtain  $X^T X$ . Use R to obtain the resulting matrix. (4 marks)
- Which of the following are orthogonal to each other

$$x = \begin{pmatrix} 1 \\ -2 \\ 3 \\ -4 \end{pmatrix} \quad y = \begin{pmatrix} 6 \\ 7 \\ 1 \\ -2 \end{pmatrix} \quad z = \begin{pmatrix} 5 \\ -4 \\ 5 \\ 7 \end{pmatrix}$$

(4 marks)

### QUESTION TWO (20 Marks)

- The following are the cholesterol contents in milligrams per packet that four laboratories obtained for 6-ounces packages of three very similar diet foods:

	Diet food A	Diet food B	Diet food C
Laboratory 1	3.4	2.6	2.8
Laboratory 2	3.0	2.7	3.1

Laboratory 3	3.3	3.0	3.4
Laboratory 4	3.5	3.1	3.7

- i. Considering diet foods as the only factor, test at level of significance  $\alpha = 0.05$  whether the difference among the three samples means can be attributed to chance (5 marks)
  - ii. Perform a two way analysis of variance and test the null hypothesis concerning the diet foods and the laboratories at 0.05 level of significance (5 marks)
- b) Using multivariate analysis of variance, analyse the following data for the concentration of three amino acids in centipede hemolymph (mg/100gl), asking whether the mean concentration of each is the same in male and female: (using the any three; Wilk's, Pillai's trace, Lawley-Hotelling trace, Roy's maximum root, or Hotelling T<sup>2</sup>) (10 marks)

Male			Female		
alanine	Aspartic acid	Tyrosine	alanine	Aspartic acid	Tyrosine
7	17	19.7	7.3	17.4	22.5
7.3	17.2	20.3	7.7	19.8	24.9
8	19.3	22.6	8.2	20.2	26.1
8.1	19.8	23.7	8.3	22.6	27.5
7.9	18.4	22	6.4	23.4	28.1
6.4	15.1	18.1	7.1	21.3	25.8
6.6	15.9	18.7	6.4	22.1	26.9
8	18.2	21.5	8.6	18.8	25.5

### QUESTION THREE (20 Marks)

The data set KE-Arrests contains a hypothetical/simulated statistics, in arrests per 100000 residents for assault, murder, and rape in each of the 47 counties in the year 2018. Also given is the percent of the population living in urban areas. Use R to solve this question.

- a) Obtain the variables summary statistics (means and standard deviations) (2 marks)
- b) What is the proportion of missing data in each of the variables (assault, murder, rape, and urbanpop) (2 marks)
- c) Develop a correlation matrix involving the above variables. Provide, the matrix. Also, indicate which two variables have strongest correlation. (3 marks)

- d) With *prcomp* function, develop a principal component analysis to the hypothetical KEarrest data. Give a rotation summary of the principal components. (4 marks)
- e) Give the contributions of the components in terms of the amount of variance explained. Discuss the result. (3 marks)
- f) Sketch a scree plot for the PCA with x-axis denoting principal component and y-axis denoting proportion of variance explained. (2 marks)
- g) Create 2 principal components and analyse performance of various counties with regards to the variables (Hint: use *biplot* function in R) (4 marks)

**QUESTION FOUR (20 Marks)**

An ecologists measured  $x_1$ =the tail length (in mm) and  $x_2$ =wing length (in mm) for a sample of 45 female hook-billed kites (single multivariate sample). The data is given in the table below

x <sub>1</sub>	x <sub>2</sub>	x <sub>1</sub>	x <sub>2</sub>	x <sub>1</sub>	x <sub>2</sub>
191	284	186	266	173	271
197	285	197	285	194	180
208	288	201	295	198	300
180	273	190	282	180	272
180	275	209	305	190	292
188	280	187	285	191	286
210	283	207	297	196	285
196	288	178	268	207	286
191	271	202	271	209	303
179	257	205	285	179	261
208	289	190	280	186	262
202	285	189	277	174	245
200	272	211	310	181	250
192	282	216	305	189	262
199	280	189	274	188	258

Required:

- i. Is the bivariate normal distribution a viable population model for this data? Explain (2 marks)

- ii. Obtain variance-covariance matrix (2 marks)
- iii. Convert the var-cov matrix to a correlation matrix, calculate the eigen values and eigen vectors and verify that the eigen vectors are orthogonal (4 marks)
- iv. Obtain the quadratic form of the covariance matrix (4 marks)
- v. Suppose it is known that  $\mu_1 = 190mm$  and  $\mu_2 = 275mm$  for males birds, are these plausible values for  $\mu_1$  and  $\mu_2$  for female birds? Explain (4 marks)
- vi. Find the 95% confidence ellipse for the population mean  $\mu_1$  and  $\mu_2$ . (4 marks)

**QUESTION FIVE (20 Marks)**

Use the data provided below relating to the production of plastic film reported in Krzanowski (2000). Tear, gloss and opacity are measures of manufactured films. The information on the rate of extrusion, and the amount of additive used are provided as experimental factors.

production of plastic films			experimental units	
tear	gloss	opacity	rate	additive
6.5	9.5	4.4	Low	Low
6.2	9.9	6.4	Low	Low
5.8	9.6	3.0	Low	Low
6.5	9.6	4.1	Low	Low
6.5	9.2	0.8	Low	Low
6.9	9.1	5.7	Low	High
7.2	10.0	2.0	Low	High
6.9	9.9	3.9	Low	High
6.1	9.5	1.9	Low	High
6.3	9.4	5.7	Low	High
6.7	9.1	2.8	High	Low
6.6	9.3	4.1	High	Low
7.2	8.3	3.8	High	Low
7.1	8.4	1.6	High	Low
6.8	8.5	3.4	High	Low
7.1	9.2	8.4	High	High
7.0	8.8	5.2	High	High
7.2	9.7	6.9	High	High

7.5	10.1	2.7	High	High
7.6	9.2	1.9	High	High

- a) Conduct a multivariate analysis of variance and obtain the summary statistics with R (4 marks)
- b) Obtain three matrices of interest in MANOVA
  - i. Total SSP (T) (4 marks)
  - ii. Between – group SSP (W) (3 marks)
  - iii. Within-group SSP (W) (3 marks)
  - iv. Wilk’s Lambda  $\frac{|W|}{|T|}$  (3 marks)
  - v. Examine the various group means (3 marks)