JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS
SPECIAL RESIT 2020/2021 ACADEMIC YEAR

SCHOOL OF MATHEMATICS, ACTUARIAL SCIENCES BPS

SEMESTER ONE, FIRST YEAR EXAMINATIONS for BSc/BEd

SUPPLEMETARY/SPECIAL

SMA103: Linear Algebra 1

Nov, 2020 **Time: 2hrs**

INSTRUCTIONS

Answer **Question1** and **two** other questions

Show all the necessary working

Question1 [30marks] Compulsory

(a) Define the vector subspaces H_1, H_2 of vector space R^3 by , $H_1 = \{(x, y, z) : x + 2y + 2z = 0\}$,
$H_2 = \{(x, y, z) : 2x + 2y - 81z = 0\}$.

(i) Verify that both H_1, H_2 do contain the zero vector. [6 marks]

(ii) Find bases for H_1, H_2. [6 marks]

(b) Suppose the mapping $L : R^3 \to R^3$ with $L\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x - y - 8z \\ x + 2y + z \\ 6z \end{bmatrix}$

(i) Show that L is linear. (ii)Determine ker (L) and Im(L). [9 marks]

(c) Given the system of linear equations

$2x + y = 700$

$5x + 3y = 20$

(i) express it in the matrix form $A\underset{\sim}{X} = \underset{\sim}{b}$

(ii) apply the elementary matrix row reduction operations on the associated augmented matrix;

$A : I : \underset{\sim}{b}$ to reduce to the final form $I : \grave{A} : \underset{\sim}{\grave{b}}$ where I is the two by two identity matrix. Compute matrix products $A\grave{A}$, $\grave{A}A$ and hence obtain A^{-1} and $\underset{\sim}{X}$. [9 marks]

Question2 [20marks]

(a) Given matrix $M = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$

(i) Show that $M^2 = 4I_{4\times 4}$ and hence find M^{-1}, the inverse of M .

(ii) Show that the following vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$

are linearly independent. [11 marks]

(b) Suppose $T:[x,y,z] \to [x, x-2y, 2y]$. Construct matrix A of linear mapping T with respect to the standard ordered basis for basis for R^3. [9 marks]

Question3 [20marks]

(a) Without using direct computation , show that $\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ are eigenvectors of

matrix $A = \begin{pmatrix} 1 & -4 & -4 \\ 8 & -11 & -8 \\ -8 & 8 & 5 \end{pmatrix}$. Give the associated eigenvalues λ_1, λ_2, λ_3 of this matrix.

Verify that $trace(A) = \lambda_1 + \lambda_2 + \lambda_3$ [12 marks]

(b)Verify that matrix $B = A^t$ [8 marks]

has same eigenvalues λ_1, λ_2, λ_3

Question4 [20marks]

Define a linear mapping T from vector space X into vector space Y i.e. $T : X \to Y$

(a) Explain what is meant by (i)kernel of T (ii)image of T (iii)rank of T (iv) nullity of T

[8 marks]

(b) State the relationship between dimension of kernel of T and rank of T [2marks]

(c) For matrix. $M = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ determine adjoint of M and hence state M^{-1} [10marks]

Question5 [20marks]

Let $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$ be a matrix of linear transformation T .

(a) Determine kernel of T [6marks]

(b) Determine range of T [7marks]

(c) State nullity and rank of T [7marks]