JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

SPECIAL RESIT 2020/2021 ACADEMIC YEAR

RESIT (MAIN)

COURSE CODE: SMA 304

COURSE TITLE: Group Theory

EXAM VENUE: **STREAM: (EDUCATION)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 marks)

a) Define the following terms as used in Group Theory

- i) A group. (4 marks)
- ii) A homomorphism from a group $(G,*)$ to another group $(H,\dagger)$. (2 marks)
- iii) A subgroup of a group $(G,*)$. (2 marks)

b) Determine whether the set $(\mathbb{R},+)$ is a group or not. (5 marks)

c) Let $(G,*)$ be a group. Then prove that for each element $x \in G$, the identity e in $(G,*)$ is unique. (4 marks)

d) Let $(G,*)$ be a group. If $a, b, c \in G$ with $a * b = c * b$, then show that $a = c$. (4 marks)

e) Explain why the following subset of $\mathbb{Z}$ is not a subgroup of $(\mathbb{Z},+)$:

- i) The set $\{-2,-1,0,1,2\}$. (1 marks)
- ii) The set $\{n \in \mathbb{Z}: n \geq 0\}$ of all non-negative integers. (2 marks)

f) If n is a positive integer, then show that the set $n\mathbb{Z} := \{nk: k \in \mathbb{Z}\}$ of all multiples of n is a subgroup of $(\mathbb{Z},+)$. (4 marks)

g) Determine the order of the following

- i) $(\mathbb{Z}_4,+)$. (1 mark)
- ii) $(\mathbb{Z},+)$. (1 mark)

QUESTION TWO (20 marks)

a) Fill in the missing parts of the Cayley table of a group given below. (4 marks)

$*$	a	b	c
a		a	
b			
c	b		

b) Determine the identity element of the group in a) above. (1 mark)

c) Determine the inverse of each element in the group in a) above. (4 marks)

d) Let $f: G \to H$ be a homomorphism from a group $(G,*)$ and $(H,\dagger)$. Define

- i) The image of f. (2 marks)
- ii) The kernel of f. (2 marks)

e) Use the subgroup test to prove that the $\mathrm{Im}(f)$ is a subgroup of $(H,\dagger)$. (7 marks)

QUESTION THREE (20 marks)

a) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 4 & 2 \end{pmatrix}$ be permutations.

i) Compute $\alpha \circ \beta$ giving your answer both in matrix and cycle notation. (7 marks)

ii) Determine α^{-1} giving your answer both in matrix and cycle notation. (3 marks)

iii) Draw the graphs for α and β. (6 marks)

b) Write the following permutation $(1,2,3)(4,6,8)$ in matrix notation. (2 marks)

c) State the First Isomorphism Theorem. (2 marks)

QUESTION FOUR (20 marks)

a) Define the term subgroup S of a group $(G,*)$. (2 marks)

b) Consider the cyclic group $(\mathbb{Z}_4, +)$.

i) Draw the Cayley table of the cyclic group, $(\mathbb{Z}_4, +)$. (5 marks)

ii) Determine the identity element of the group. (1 mark)

iii) Determine the inverse of each element of the group. (4 marks)

iv) Determine the subgroups of the cyclic group, $(\mathbb{Z}_4, +)$. (4 marks)

v) Determine, in the cyclic group $(\mathbb{Z}_4, +)$, the order of the following elements

I) 2. (2 marks)

II) 3. (2 marks)

QUESTION FIVE (20 marks)

a) Let $(G,*), (H,\odot)$ and $(M,\dagger)$ be groups, $f: G \to H$ and $g: H \to M$ be homomorphisms.

i) Then prove that $f \circ g: G \to M$ is also a homomorphism. (5 marks)

ii) If f is an isomorphism, show that $f^{-1}: H \to G$ is also an isomorphism. (5 marks)

iii) If f and g are isomorphisms, show that $f \circ g: G \to M$ is also an isomorphism. (5 marks)

b) Let $(G,*)$ be a group and H a subgroup. Then show that H is normal if and only if for each $x \in G$, the left coset is equal to the right coset. (5 marks)