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YEAR THREE SEMESTER TWO EXAMINATION (Special Resit)2020
SMA 312 : Operations Research I

INSTRUCTION: Answer Question ONE and ANY other TWO questions.

QUESTION ONE (COMPULSORY)-30 MARKS

Define the following terms used in Operations Research

a) (i) Objective function (In linear programming)
   (ii) Optimization
   (iii) Decision Variable
   (iv) Feasible solution
   (v) Slack variable
   (vi) Artificial variable
   (vii) Sensitivity Analysis (14 marks)

b) . Use Gauss Jordan method to solve the set of simultaneous equations

$$2x_1 + 3x_2 + x_3 - 17 = 0$$
$$-4x_1 + x_2 - 3x_3 + 11 = 0$$
$$-3x_1 + 2x_2 - 2x_3 = -3.5$$

(6 marks)

c) A Printing firm owner employs both skilled workers and apprentices. The facilities available cannot allow more than 9 employees altogether. The firm must maintain at least an output of 30 units of printing work daily. On average a skilled worker does 5 units of printing and an apprentice 3 units of work daily. The Apprentice's arrangement demands that the printing firm should employ not more than 7 skilled workers to every 2 apprentices. The workers union however forbids the firm to employ less than 2 skilled men to each apprentice. The firm pays each skilled worker shs 80 per day and an apprentice shs 40.

   (i) Formulate a linear programming problem hence state any three suitable methods that can be used in optimization. (7 marks)
   (ii) .Form a dual of the primal problem in (i) above. (2 marks)

QUESTION TWO (20 MARKS)

1. a) Consider the linear programming problem

Maximise $Z = c_1x_1 + c_2x_2 + c_3x_3$

Subject to

$$\begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} x_1 + \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix} x_2 + \begin{pmatrix} a_{13} \\ a_{23} \end{pmatrix} x_3 + \begin{pmatrix} 1 \\ 0 \end{pmatrix} s_4 + \begin{pmatrix} 0 \\ 1 \end{pmatrix} s_5 = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$x_1, \ldots\ldots\ldots x_5 \geq 0$$

The optimal tableau is as follows

Basis	x_1	x_2	x_3	S_4	S_5	B
Z	3	0	0	0	4	20
X_3	1	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$
X_2	$\frac{1}{2}$	1	0	-1	2	2

i) Find the values of

$$\begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix}, \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix}, \begin{pmatrix} a_{13} \\ a_{23} \end{pmatrix} and \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

ii) Find the values of C_1 , C_2 and C_3 (9 marks)

iii) Find the range of values within which C_2 can fall without changing the optimal solution (3 marks)

b) Use the two phase method to solve the LP problem below

Minimize $Z = 2x_1 + 9x_2 - 4x_3$

Subject to

$$2x_1 + 3x_2 + 4x_3 \leq 8$$

$$x_1 + 6x_2 - 4x_3 \geq 24$$ (8 marks)

$$x_1, x_2, x_3 \geq 0$$

QUESTION THREE(20 MARKS)

a) Three types of coupling units are produced by a firm. The times required for machining, polishing and assembling a unit of each type are included in the following table

	Type in hours per unit			
Type of unit	Machining	Polishing	Assembling	Profit per unit
A	2	4	1	30
B	4	2	1	40
C	3	1	2	50
Available time hours/week	80	48	40	

Advise the firm on how many of each type of coupling to produce in order to maximize profit. (14 marks)

b) Suppose in (a) above the available time in hours per week changed from 80 to 60 for machining, 48 to 56 for polishing and 40 to 30 for assembling. Would your original advise on levels of optimal production still stand. What would be the new optimal solution. (6 marks)

QUESTION FOUR (20 MARKS)

a) By appropriately assigning an artificial variable, use any relevant Linear programming technique to

Minimise $Z = -3x_1 + 4x_2$

Subject to

$$x_1 + 3x_2 \leq 54$$
$$3x_1 + x_2 \leq 34$$
$$-x_1 + 2x_2 \geq 12$$
$$x_1, x_2 \geq 0$$

(10 marks)

b) A company has three ware houses A, B and C and four stores W, X, Y and Z. The warehouses have altogether a surplus of 1600 units of a given commodity as follows

A	500
B	700
C	400

The four stores together need a total of 1500 units of the commodity as follows

W	300
X	500
Y	700
Z	100

The cost of transporting one unit in Ksh from each warehouse to store is shown in the table below

	W	X	Y	Z
A	500	100	600	400
B	600	800	900	200
C	250	820	525	780

Determine which method would be cheaper and by how much between Vogel's approximation and the least cost cell method as far as the cost of transport is concerned (10 marks)

QUESTION FIVE (20 MARKS)

a) A firm produces two types of perfume A and B. It can produce 8 bottles of perfume every minute. The quantity of perfume A must be less than thrice the quantity of perfume B while twice the quantity of perfume A must exceed that of perfume B. Additional information is as follows.

	Production cost per bottle(kshs)	Labourers per bottle	Profit per bottle
A	300	1	70
B	100	4	50
Available	1800	20	

i) Develop a linear programming model based on the information above.

ii) Using graphical method and an isoprofit (search) line advise the firm on the number of bottles of each perfume to produce in order to maximize profit. (12 marks)

b) Use the Dual simplex method to solve the following LP problem

Minimize $Z = 2x_1 + 4x_2$

Subject to

$2x_1 + x_2 \geq 4$

$x_1 + 2x_2 \geq 3$

$2x_1 + 2x_2 \leq 12$

$x_1, x_2 \geq 0$

(8 marks)