



**JARAMOGI OGINGA ODINGA UNIVERSITY OF  
SCIENCE AND TECHNOLOGY**

**UNIVERSITY EXAMINATIONS  
SPECIAL RESIT 2020/2021 ACADEMIC YEAR**

*SCHOOL OF MATHEMATICS, ACTUARIAL SCIENCES BPS*

**SEMESTER TWO, SECOND YEAR EXAMINATIONS for BSc/BEd**

***SUPPLEMENTARY/SPECIAL***

**SMA414 Fourier Analysis**

**Nov, 2020**

**Time: 2hrs**

**INSTRUCTIONS**

Answer **Question1** and **two** other questions

**Show all the necessary working**

**QUESTION 1 (30 MARKS)**

a) Find the  $\lim_{x \rightarrow 0} \left\{ \frac{\cos(x^2) - 1 + x^2}{x} \right\}$  (5 marks)

b) Determine whether the given functions are even, odd or neither

i)  $f(x) = \sin\left(\frac{n\pi x}{L}\right)$  on  $-L \leq x \leq L$

ii)  $f(x) = \cos\left(\frac{2n\pi x}{L}\right)$  on  $-L \leq x \leq L$

iii)  $f(x) = x^2 - 11 + e^{-2x}$  on  $-L \leq x \leq L$  (9 marks)

c) Compute the Maclaurin series as far as  $x^6$  term for the following functions

i)  $\frac{\sin(x)}{x}$

ii)  $\frac{\sin(x^2)}{x^2}$  (8 marks)

d) The Fourier series of the function  $f$  defined by  $f(x) = x^2$  on the interval  $[-\pi, \pi]$  is known to be convergent.

What do you understand by

- i) period of  $f$
- ii)  $f$  is periodic
- iii) periodic extension of  $f$
- iv) Fourier coefficients of expansion

Give a sketch graph of two periodic extensions of  $f$  (8 marks)

**QUESTION 2 (20 MARKS)**

Solve the heat equation  $u_t = \frac{1}{100}u_{xx}$ ,  $0 < x < 1$ ,  $t > 0$  with the Dirichlet boundary conditions  $u(t,0) = u(t,1) = 0$ ,  $t > 0$  and initial conditions  $u(0,x) = g(x) = x$ ,  $0 \leq x \leq 1$   
(20 marks)

**QUESTION 3 (20 MARKS)**

Find the Fourier series of the function defined in pieces (piecewise constant function) by

$$f(x) = \begin{cases} 8 & 0 < x < 4 \\ -8 & 4 < x < 8 \end{cases}$$

where  $f$  is periodic with period 4. What does the series converge to at

- i)  $x = 2$
- ii)  $x = 5$  (20 marks)

**QUESTION.4 [20 marks]**

(a) Given the voltage  $v = f(t)$  volts, and  $i = F(t)$  amperes, such that  
 $v = 12.0 + 5.2 \cos wt + 2.4 \cos 2wt + 0.9 \cos 3wt + \dots + 2.7 \sin wt + 1.8 \sin 2wt + 0.2 \sin 3wt + \dots$   
 $i = 8.50 + 4.1 \cos wt + 2.0 \cos 2wt + 0.6 \cos 3wt + \dots + 3.6 \sin wt + 1.2 \sin 2wt + 0.3 \sin 3wt + \dots$   
find the average value of power  $vi$  in watts, over one cycle. [9 marks]

(b) For the function  $f(x) = \begin{cases} 12x & -5 < x < 5 \\ f(x+10) & \text{otherwise} \end{cases}$

- (i) sketch graph of  $f(x)$  over the interval  $-20 < x < 20$
- (ii) state period of  $f(x)$
- (iii) obtain Fourier series for  $f(x)$  [11 marks]

**QUESTION 5 [20 marks]**

One cycle of a periodic waveform  $y = f(x)$  of period  $2\pi$  is defined by the below data.

$x^\circ$	0	30	60	90	120	150	180	210	240	270	300	330
$y(x)$	15	20	23	20	14	8	3	4	9	12	10	11

Determine the approximate Fourier series for  $y = f(x)$  up to and including the third harmonic.

[20 marks]