JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE

SPECIAL RESIT 2020/2021 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SMA 3111

COURSE TITLE: Mathematics I

EXAM VENUE: **STREAM: HEALTHSCI, AGRI, ENGINEERING**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

<u>QUESTION ONE</u> (30 marks)

a) Define the following terms as used in set theory and give example in each case.
   i) Cardinality of a set (2marks)
   ii) Universal set (2marks)

b) Solve the equation $x^2 - 6x + 2 = 0$ by completing the square (4marks)

c) i) How many committees of 5 people can be formed from a pool of 12 people (2marks)

   ii) Use Binomial theory to determine the expansion of $(2a - 3b)^5$ (5marks)

d) Prove the identity
$$\frac{\cos\theta}{1-sin\theta} - \frac{1}{cos\theta} = tan\theta$$
(5marks)

e) Solve the equation $\log(x^2 - 3) - \log x = \log 2$ (3marks)

f) A geometric sequence has the first term as 3 and common ratio ss 2, the sequence has eight terms. Find:

   i) The last term (2marks)
   ii) The sum of the terms in the sequence (2marks)

g) Solve $sin\theta = \frac{1}{2}$ for $0 < \theta < 2\pi$ (3marks)

<u>QUESTION TWO</u> (20 marks)

a) The following table shows the distribution of marks in percentages scored by a class of forty students in a promotion examination.

Marks	20-29	30-39	40-49	50-59	60-69	70-79	80-89
Runners	6	5	7	10	5	4	3

   Use the data to compute

   i) mean (3marks)

   ii) median (4marks)

   iii) standard deviation from the above data (3marks)

b) Given $A = \{u, v, w, x\}$ and $B = \{a, b, c\}$. Let R be the following relation from A to B. $R = \{(u, b), (u, c), (w, b), (x, a), (x, c)\}$
   i) Determine the arrow diagram of R (2marks)
   ii) Find the inverse relation R^{-1} of R (2marks)
   iii) Determine the domain and the range of R^{-1} (2marks)

c) Given that $A = \{a, b\}$ and $B = \{x, y, z\}$ Show that the $A \times B \neq B \times A$ (4 marks)

<u>QUESTION THREE</u> (20 marks)

a) i) Three numbers are in arithmetic progression. Their sum is 15 and their product is 80. Determine the 3 numbers (6marks)

ii) An oil company bores a hole 80 metres deep. Estimate the cost of boring if the cost is \$ 30 for the first metre with an increase in cost of \$ 2 per metre for each succeeding metre. (4marks)

b) During the first semester in the Department of Mathematics, JOOUST University, 18 students took SMA 101, 25 took SMA102, 23 took SMA 103 and 9 took SMA 101 and SMA 102, 10 took SMA102 and SMA 103 and 6 took SMA 101 and SMA 103. If there were 50 students and 5 students did not take any of the three courses, with the aid of the Venn diagram find how many students took

i) All 3 courses
ii) Only SMA102
iii) SMA 103 but not SMA 102
iv) SMA 101 and SMA 103 but not SMA 102.

(10marks)

<u>QUESTION FOUR</u> (20 marks)

a) Find the power set of $A = \{a, \{1,2\}\}$ (2marks)

If $\mathcal{U}$ is the universal set of all positive integers and P, Q, R are subsets such that

$P = \{x: x \text{ is a prime number}\}$

$Q = \{x: x \text{ is an even number}\}$

$R = \{x: 7 < x \leq 20\}$

List the elements of:

i) $P \cap R$ (1mark)
ii) $Q^c \cap R$ (2marks)
iii) $P^c \cap (Q^c \cap R)$ (2marks)

b) Draw the Venn diagram and shade the region corresponding to $(A^c \cap B) \cap C^c$ (3marks)

c) Solve the equation $2sin^2\theta = cos\theta + 1$ for θ in the range $0^o \leq \theta \leq 360^o$ (5marks)

d) Use the remainder theorem to evaluate $f(x) = 6x^3 - 5x^2 - 4x - 17$ at $x = 3$ (5marks)

<u>QUESTION FIVE</u> (20 marks)

a) Show that the area A of an isosceles triangle whose equal sides are of length s and θ is the angle between them is $A = \frac{1}{2}s^2 \sin\theta$ (5 marks)

b) Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x^2 - 3$ and $g(x) = 4x$. Find

i)$(fog)(x)$ (3marks)

ii)$(gof)(x)$ (3marks)

c) Find the inverse of $f(x) = 2x - 3$ (3marks)

a) Prove the following distributive law of set operations:

$F \cap (G \cup H) = (F \cap G) \cup (F \cap H)$ (6 marks)