JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF   SCIENCE IN ACTUARIAL SCIENCE

3RD   YEAR 1ST SEMESTER 2019/2020 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SAC 305

COURSE TITLE: PENSION MATHEMATICS

EXAM VENUE: STREAM: (BSc. Actuarial)

DATE: EXAM SESSION:

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**

2. **Candidates are advised not to write on the question paper.**

3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION 1 [COMPULSORY] [30 Marks]

(a) Describe the methods of Actuarial costing as applied in valuations of pension schemes **[4 Marks]**

(b) Consider a person now aged exactly 25 whose annual salary rate is Kshs 91,920. Estimate

(i)his annual salary rate at exact age 53 **[2 Marks]**

(ii) his earnings between age 64 and 65 **[3 Marks]**

(iii) the average amount earned by him each year between exact ages 60 and 65 **[3 Marks]**

Assume that salaries are revised continuously and use the pension table with 4% p.a interest.

(c) State three options that may be granted in relation to retirement benefits **[6 Marks]**

(d)The men present value of the future contributions(of employee,employer or both) at a rate of $k\%$ of salary for a member age x with a current salary rate of SAL per annum is

$$\frac{k}{100}\frac{SAL}{\bar{s}_x}\int_0^{65-x} v^t \frac{l_{x+t}}{l_x}\bar{s}_{x+t}dt$$

Using approximation and appropriate commutation functions, show that the m.p.v is

$$\frac{k}{100}\frac{SAL}{\bar{s}_x D_x}{}^s\bar{N}_x$$

[6 Marks]

(e) Define the following pension benefit valuation models

(i) Emerging Cashflow Model [**2 Marks**]

(ii) Benefit Event Model [**2 Marks**]

(iii) Commutation Functions Model [**2 Marks**]

QUESTION 2[20 MARKS]

(a) Using the following assumptions and data :

$$i = 10\%p.a$$

$$e = 8\%p.a$$

$$A = 60\%(\text{ie accrual rate is 60ths})$$

$$R = 65$$

$$a'_R == 12$$

Calculate the AL and SCR under the Attained Age (AA) method for the following members individually;

(i) 25 year old, no past service, salary Kshs.20, 000 [**4 Marks**]

(ii) 40 year old, 15 years past service, salary Kshs.15, 000 [**4 Marks**]

(iii) 55 year old, 30 years past service, salary Kshs.30, 000 [**4 Marks**]

The same earnings definition is used for benefit purposes. You may assume for simplicity, that contributions are paid in arrears.

(b) Calculate the AASCR and AAAL for the scheme that contains only these three members.Compare the SCR with the average of the three individual SCRs and comment on your answer. [**8 Marks**]

QUESTION 3[20 MARKS]

You are given the following data concerning a final salary pension scheme (where pensionable salary equals salary). All the data relates to the current membership except where stated.

Value of all benefits accruing in the next year based on current salary - 9,000,000

Value of all benefits accruing in the next year based on salary rate at the end of this year -9,600,000

Value of all benefits accruing in the next year based on fully projected earnings -12,000,000

Value of all benefits accruing over the future working lifetime based on fully projected earnings -117,000,000

Current salary roll-103,000,000

1% of salary over the next year -1,000,000

1% of salary over the future working lifetime -9,000,000

1% of salary over the next year -1,000,000

1% of salary for a new single entrant -10,000

Value of accrued benefits based on current salary-240,000,000

Value of accrued benefits salary at the end of the year-256,000,000

Value of accrued benefits allowing for fully projected earnings-300,000,000

Value of benefits in respect of deferred pensioners-50,000,000

Value of benefits in respect of pensioners-150,000,000

Calculate Standard Contribution Rate(SCR) and the Actuarial Liability

(AL) with respect to the following methods

(a) Attained Age (AA)Method   [**5 Marks**]

(b) Entry Age (EA) Method   [**5 Marks**]

(c) Projected Unit (PU) Method   [**5 Marks**]

(d) Current Unit (CU)   [**5 Marks**]

QUESTION 4[20 MARKS]

(a) A pension scheme provides a pension of $\frac{1}{45}$ of final pensionable salary for each year of service, with a maximum of $\frac{2}{3}$ of final pensionable upon retirement at age 65. Final pensionable salary is defined as average annual salary over the 3 years immediately preceding retirement. A member is now aged 45 exactly and has 14 years of past service. He earned 400,000 in the previous 12 months. Calculate the expected present value now of this member's total pension on retirement, using the symbols defined in, and assumptions underlying, the Formulae and Tables for Actuarial Examinations.   [**7 Marks**]

(b) A pension scheme provides an ill-health retirement pension of $\frac{1}{60}$ of Final Pensionable salary for each year of company service, with fractions of a year to count proportionately, subject to a maximum of Final Pensionable salary. Retirement die to ill-health may take place at any age before 65. Final Pensionable salary is defined as the average annual salary over the three year period preceding retirement.

Derive commutation functions to value the ill-health pension for a mem-

ber aged exactly 25 who has completed exactly 5 years company service to date. Define carefully all the symbols that you use.   [**14 Marks**]

QUESTION 5[20 MARKS]

(a) What are the assumptions underlying the discounted value of assets in the asset valuation model?   [**6 Marks**]

(b) Three equities E_1, E_2 and E_3 have market values of 1,000,000 each. The dividends payable in one year's time will be 70,000 for E_1, 50,000 for E_2 and 30,000 for E_3. The valuation assumptions used are 10% p.a interest and 5% dividend growth.

(i)Using the assumptions, and assuming dividends are payable annually in arrears, calculate the discounted proceeds value of each asset.   [**7 Marks**]

(ii)A student has suggested that more sensible values are obtained if assets are valued using an assumed growth rate of 10% minus the current prospective dividend yield for each holding (e.g using a growth rate of 3% for E_1, i.e 10%-7%). Ca;culate the values using this approach and comment on your result.   [**7 Marks**]