



**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN  
ACTUARIAL SCIENCE**

**4<sup>TH</sup> YEAR 1<sup>ST</sup> SEMESTER 2019/2020 ACADEMIC YEAR**

**MAIN REGULAR**

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**COURSE CODE: SAC 403**

**COURSE TITLE: ACTUARIAL LIFE CONTINGENCIES II**

**EXAM VENUE:**

**STREAM: (BSc. Actuarial)**

**DATE:**

**EXAM SESSION:**

**TIME: 2.00 HOURS**

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**Instructions:**

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**QUESTION 1 [COMPULSORY] [30 Marks]**

(a) What is a reserve? Distinguish between a prospective reserve and a retrospective reserve.

[6 Marks]

(b) Write down in the form of symbols, and also explain in words, the expressions “expected death strain” and “actual death strain”, hence or otherwise the “mortality profit” for a portfolio of policies. [8 Marks]

(c) At the start of a particular year a life insurance company had a portfolio of 5,000 pensioners, all aged exactly 60, who each receive an income of Kshs.10,000 per annum, paid annually in arrears.

The company holds net premium reserves, calculated using PFA92C20 mortality and 4% p.a interest. During that year, 9 pensioners died. Calculate the mortality profit or loss and comment on your answer.

[6 Marks]

(d)(i) A ten year regular premium term assurance policy is issued a group of lives aged 40. The sum assured 20,000 and is payable at the end of the year of death. Write down an expression for the retrospective reserve immediately before the 6th premium is due, assuming that reserves are calculated on the same basis as premium  $P$ . [5 Marks]

(ii) A temporary annuity of 3000 payable annually in arrears for a term of 10 years was purchased one year ago by a life then aged exactly 60 by payment of a single premium. Show algebraically that the current retrospective and current prospective net reserves are equal assuming that the

pricing and reserving bases are the same.

[7 Marks]

### QUESTION 2[20 MARKS]

A life insurance company issues 20-year temporary assurance policies to lives aged 45. The sum assured, which is payable immediately on death is Kshs. 400,000 for the first 10 years and Kshs. 100,000 thereafter. Level premiums are payable in advance for 20 years, or until earlier death. (Take the benefit payable under this temporary assurance as  $100.000\bar{A}_{45:20}^1 + 300.000\bar{A}_{45:10}^1$ )

The premium basis is:

Mortality: AM92 Ultimate

Interest: 4% per annum

Expenses: nil

(a) Calculate the annual premium payable.

[12 Marks]

(b) Find the net premium reserve ten years after the commencement of the policy, immediately before the payment of the eleventh premium, assuming the reserving basis is the same as the premium basis.

[8 Marks]

**QUESTION 3[20 MARKS]**

(a) In the context of net premium and reserves, state the conditions necessary for the equality of prospective and retrospective reserve.

(b) Give formulae for the following net premium reserves in terms of other monetary functions:

$${}_tV_{\overline{x:n}|}$$

and

$${}_t\bar{V}(\bar{A}_{\overline{x:n}|})$$

by

(i) the prospective method and

[6 Marks]

(ii) the retrospective method

[6 Marks]

(c) Show that

$${}_1W_{\overline{n}|} = \frac{1}{\ddot{a}_{\overline{n}|}}$$

[8 Marks]

**QUESTION 4[20 MARKS]**

A life insurance company issues a non-profit assurance policy for a term of  $n$  years to a life aged  $x$  exact.

For  $t = 1, 2, \dots, n$  :

- The level annual premium payable at the start of year  $t$  is  $P$ .
- The expense at the start of the policy year  $t$  is  $E_t$ .

- The benefit payable at the end of the  $t^{\text{th}}$  policy year on death, surrender and survival are  $D_t$ ,  $B_t$  and  $S_t$  respectively.
- The rate of interest earned on net cash flows during the  $t^{\text{th}}$  policy year is  $i_t$
- The dependent rates of mortality and surrender at age  $x+t$  are  $(aq)_{x+t}^d$  and  $(aq)_{x+t}^w$  respectively.

(a) Assume that the insurance company does not set up a reserve for the policy. Write down an expression for  $(CF)_t$ , the accumulation to the end of the  $t^{\text{th}}$  policy year of the expected net cash flow. **[6 Marks]**

(b) Derive an expression which could be used to calculate the level annual premium that the company should charge if the company requires the expected net present value of profit on the policy to be zero assuming a risk discount rate of  $j\%$  per annum defining any notations used .

**[8 Marks]**

(c) Assume that the insurance company does set up a reserve  ${}_{t-1}V$  for the policy at the start of the  $t^{\text{th}}$  policy year. Write down an expression for the expected profit at the end of the  $t^{\text{th}}$  policy year for each policy in force at the start of that year. **[6 Marks]**

**QUESTION 5[20 MARKS]**

A special endowment policy pays a sum assured of 20,000 to a life who is currently aged exactly 57 after three years or at the end of the year of earlier death. Annual reversionary bonuses are declared at the end of each policy year, and a terminal bonus is payable at maturity only. Policies may be surrendered only at the end of each policy year. On surrender, the policyholder receives a return of premiums with interest calculated at the rate of 3% per annum. A level premium of 8,000 is paid at the start of each year. The premium basis is as follows;

- Interest- 7% per annum
- Mortality- AM92 select
- Surrender rates -15% of all policy in force at the end of the year, 5% of all policy in force at the end of year 2.
- Reversionary bonuses -6% per annum compound
- Terminal bonuses-10% of all other benefits payable at maturity
- Expenses -Initial 500, Renewal 30 at the start of year 2, 35 at the start of year 3, Termination 100 per termination
- Reserves -Net premium reserves, using AM92 ultimate mortality and 4% per annum interest

Calculate the profit signature according to the premium basis.

**[20 Marks]**