JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

4^{TH} YEAR 2^{ND} SEMESTER 2018/2019 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SAC 408

COURSE TITLE: RISK MATHEMATICS

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

(a) Define the following terms as used in actuarial mathematics;

i. efficient frontier

ii. indifference curve

iii. optimal portfolio. [3 marks]

(b) Explain the four axioms that are required to derive the expected utility theorem. [6 marks]

(c) Consider the two risky assets, A and B, with cumulative probability distribution functions:

$$F_A(w) = w$$

$$F_B(w) = w^{0.5}$$

In both cases, $0 \leq w \leq 1$.

i. Show that A is preferred to B on the basis of first-order stochastic dominance. [3 marks]

ii. Verify explicitly that A also dominates B on the basis of second-order stochastic dominance. [3 marks]

(d) State the other assumptions underlying the Black-Scholes model. [5 marks]

(e) Consider a zero-coupon corporate bond that promises to pay a return of 10% next period. Suppose that there is a 10% chance that the issuing company will default on the bond payment, in which case there is an equal chance of receiving a return of either 5% or 0%. Calculate values for the following measures of investment risk:

i. downside semi-variance

ii. shortfall probability based on the risk-free rate of return of 6%

iii. the expected shortfall below the risk-free return conditional on a shortfall occurring. [5 marks]

(f) An investor can invest in only two risky assets A and B. Asset A has an expected rate of return of 10% and a standard deviation of return of 20%. Asset B has an expected rate of return of 15% and a standard deviation of return of 30%. The correlation coefficient between the returns of Asset A and the returns of Asset B is 0.6.

i. What is the expected rate of return if 20% of an investors wealth is invested in Asset A and the remainder is invested in Asset B? [2 mark]

ii. What is the standard deviation of return on the portfolio if 20% of an investors wealth is invested in Asset A and the remainder is invested in Asset B? [2 marks]

QUESTION TWO

(a) Let S_t be a geometric Brownian motion process defined by the equation $S_t = \exp(\mu\, t + \sigma\, W_t)$, where t W is a standard Brownian motion and m and s are constants.

   i. Write down the stochastic differential equation satisfied by $X_t = \ln\, S_t$. [2 marks]

   ii. By applying Itos Lemma, or otherwise, write down the stochastic differential equation satisfied by S_t . [3 marks]

   iii. The price of a share follows a geometric Brownian motion with $\mu = 0.06$ and $\sigma = 0.25$ (both expressed in annual units). Find the probability that, over a given one-year period, the share price will fall.
   [3 marks]

(b) Two investments are available. A risk-free investment B that returns 1%, and an investment A whose return is given by:

$$R_A = \begin{cases} -1\%, & \text{prob}\, 0.5; \\ 3\%, & \text{prob}\, 0.5. \end{cases}$$

   i. Explain why Asset B must be second-order stochastically dominant over Asset A in terms of investors and utility functions. [2 marks]

   ii. Verify numerically the second-order stochastic dominance expressed in part (i). [2 marks]

(c) Show that the following utility functions have constant relative risk aversion co-efficient

   i. $u(x) = \ln\, x$ [3 marks]

   ii. $u(x) = \alpha\, x^{\alpha}$ [3 marks]

QUESTION THREE

(a) An investor is contemplating an investment with a return of Ksh R, where:

$$R = 250,000 - 100,000N$$

and $N \sim\, N[1, 1]$ random variable. Calculate each of the following measures of risk:

   i. variance of return [3 marks]

   ii. downside semi-variance of return [3 marks]

   iii. shortfall probability, where the shortfall level is 50,000 [3 marks]

   iv. Value at Risk at the 5% level [3 marks]

(b) Claims arrive according to a Poisson process. Individual claim sizes are independent with density:

$$f(x) = xe^{-x},\ x > 0$$

and the insurer uses a premium loading factor of θ.

i. Derive the equation for the adjustment coefficient for this process. [2 marks]

ii. If $\theta = 0.4$, calculate the adjustment coefficient, and determine an upper bound for the probability of ultimate ruin if the initial surplus is 50. [4 marks]

QUESTION FOUR

(a) Claims arrive in a Poisson process rate , and the claim severity distribution has mean and moment generating function $M(t)$. The premium income per unit time is c where $c > \lambda\mu$.

i. Write down an equation satisfied by the adjustment coefficient. [1 mark].

ii. Derive the adjustment coefficient in terms of λ, μ and c, when the claims are exponentially distributed with mean. [2 marks]

iii. Calculate the adjustment coefficient R_{exp} when $\mu = 100$ and the premium loading factor is 25%. [3 marks]

iv. State Lundberg's inequality for the probability of ruin with initial capital μ. [2 marks]

v. Determine and comment on the effect on R_{exp} if the mean claim size is increased but the premium loading factor remains the same. [2 marks]

vi. Determine and comment on the effect on R_{exp} if instead the premium loading factor is increased but the mean claim size stays the same. [1 mark]

(b) Consider security A, which has a standard deviation of 4%. If the standard deviation of the market return is 5%. The correlation between $A's$ return and that of the market is 0.75. The risk-free rate is 5% and the expected return on the market is 10%. Calculate

i. the beta of security A. [3 marks]

ii. security $A's$ expected return. [3 marks]

(c) An insurer knows from past experience that the number of claims received per month has a Poisson distribution with mean 15 and that claim amounts have exponential distribution with mean 500. The insurer uses a security loading factor of 30%. calculate the insurer's adjustment coefficient and the probability of ruin if the initial surplus was 1000. [3 marks]

QUESTION FIVE

(a) The Capital Asset Pricing model is assumed to hold in a particular investment market.The total return on a unit invested in asset A in this market has mean 1.15 and standard deviation 0.10. The return on a unit invested risk free is 1.05 and the expected return on a unit invested in the market portfolio is 1.08. You are given that A is an efficient portfolio.

i. Find the equation for the capital market line. [3 marks]

ii. Calculate the standard deviation of the return on the market portfolio. [3 marks]

iii. Calculate the β for asset A. [5 marks]

(b) State the assumptions of CAPM [5 marks]

(c) Investor A has an initial wealth of 100 and a utility function of the form $U(w) = \ln(w)$. Investor Z offers her a return of -18% or $+20\%$ with equal probability.

i. What is her expected utility if she invests nothing in investment Z? [2 marks]

ii. What is her expected utility if she invests entirely in investment Z? [2 marks]