JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF   SCIENCE ACTUARIAL

1ST   YEAR 1ST SEMESTER 2018/2019 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SAC 101

COURSE TITLE: PRINCIPLES OF ACTUARIAL SCIENCES

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

a) Briefly explain the seven principles of insurance. [7 marks]
b) Calculate the present value of 100 over 10 years at the following rates of interest or discount
    i.   At an effective interest rate of 5% p.a convertible monthly. [2 marks]
    ii.   At an effective discount rate of 5% p.a convertible monthly.[2 marks]
    iii.   At a force of interest rate of 5% p.a convertible monthly. [2 marks]
c) At the end of each month, a saver deposited sh. 100 into a savings account that paid 6% compounded monthly. How much would he have at the end of 10 years? [5 marks]
d) In a certain country, censuses are held on 30th june each year. The enumerated male population on 30th june, 1993, 1994 and 1995 at ages 30, 31, 32 and 33 last birthday was as follows.

Age	Year		
	1993	1994	1995
30	306421	303606	307412
31	300112	305169	302121
32	299408	299168	304003
33	299133	298977	299015

Estimate the central exposed to risk from 1993 to 1995 at each ages 30, 31, 32 and 33 last birthday using
    i.   The repeated mid-point rule. [3 marks]
    ii.   The trapezoidal method. [3 marks]
e) Suppose that the life table function is given by the formular
$$l_x = (100 - x)^{0.5}$$
Compute the following probabilities q_{49} and $_{2|}q_{41}$. [4 marks]

QUESTION TWO

a) A customer is offered an investment where interest is calculated according to the following force of interest
$$\delta_t = \begin{cases} 0.02t & 0 \le t \le 3 \\ 0.045 & t > 3 \end{cases}$$

The customer invests sh. 1000 at time t=0. What nominal rate of interest compounded quarterly is earned over the first four year period. [4 marks]

b) Jane , Lori and Lucy each borrow a loan of sh. 5000 for five years at a nominal interest rate of 12% compounded semi-annually.
Jane has interest accumulated over the five years and pays all the interest and principal in a lump sum at the end of five years.
Lori pays interest at the end of every six-month period as it accrues and the principal at the end of the five years.
Lucy repays her loan with 10 level payments at the end of every six-month period.
Calculate the amount of interest paid on the three loans.　　[8 marks]

c) Compare and contrast the graduation of reference to a standard table method and mathematical formula of graduation.　　[8 marks]

QUESTION THREE

a) Discuss the main types of life insurance.　　[5 marks]
b) Bruce and Robbie each open up new bank accounts at time 0. Bruce deposits sh. 100 into his bank account and Robbie deposits sh. 50 into his. The amount of interest earned in Bruce's account during the 11th year is equal to x. The amount of interest earned in Robbie's account during the 17th year is also equal to x. Calculate x.　　[5 marks]
c) Suppose you deposit sh. 100 at the end of year one, 200 at the end of year two and 300 at the end of year three. Find the present value if the annual interest rate is 10%.　　[3 marks]
d) Differentiate between the nominal rate of interest and the effective rate of interest.　　[4 marks]
e) Explain precisely what is meant by $_{3|5}q_{50}$.　　[2 marks]

QUESTION FOUR

a) Suppose you are offered an investment that will pay you sh. 200000 in year one, sh. 400000 in year two, sh. 600000 in year three and sh. 800000 in the

final year. In this investment you earn an interest rate of 12%. What is the most you should pay for? [5 marks]

b) You can buy a security now for sh.1000 and it will pay you sh. 1191 three years from now. What annual rate of return are you earning? [3 marks]

c) Given that a savings bank charges a nominal interest rate of 10% compounded monthly. What is the corresponding effective annual rate of interest? [2 marks]

d) Suppose that $\mu_{x+0.5}(\alpha) = \alpha.\mu_{x+0.5}^T$. Find a simple formular for $\hat{\alpha}$, the Maximum Likelihood Estimator of the true value of $\alpha_1 = \alpha$. [4 marks]

e) Discuss the three methods of investment appraisal techniques. [6 marks]

QUESTION FIVE

a) Consider the following extract of a life table:

x	l_x	q_x	p_x	d_x
40	24983			
41	24541			
42	24175			
43	23880			
44	25656			
45	23495	-	-	-

Use the table to answer the following questions;

i. Complete the table. [3 marks]

ii. Find the probability that (41) survives for another two years. [1 mark]

iii. Find the probability that (40) dies between ages 42 and 45. [2 marks]

iv. Find the probability that (30) will die within two years. [2 marks]

b) Discuss any five importance lifetables in actuarial science. [5 marks]

c) Let P_x^0 and P_x^1 denote the number of lives present and aged x last birthday at the beginning and end respectively of a year-long investigation and let θ_x denote the number of deaths at age x last birthday during the year. Show that $E_x = \frac{1}{2}[P_x^0 + P_x^1 + \theta_x]$. [5 marks]