QUESTION ONE

(a) Explain the following concepts

    i. Annuity certain payable in advance.

    ii. Cash flow

    iii. Insurance lapse

    iv. Guaranteed Endowment

    v. Force of mortality.   [5 marks]

(b)   i. Explain whether and why the insurer will have to charge a 'higher or lower premium' if
    (1) Interest rates increase
    (2) Mortality rates decrease.   [4 marks]

    ii. Explain in words the model;

$$\ddot{a}_{\overline{n}|} = (1+i)a_{\overline{n}|} = 1 + a_{\overline{(n-1)}|}$$

    [1 mark]

    iii. A. Give and explain who would wish to buy a pure endowment factor, giving reasons for its popularity.   [3 marks]

       B. XYZ takes out a loan of 100,000 dollars to purchase a property. The loan is to be repaid overa 25 year period by amking equal annual payments in arrears. Find;

       C. The annual payment if the APV is 10 percent.   [3 marks]

       D. If XYZ sells his property and repays the outstanding amount of the loan at the time that the tenth annual payment is done (but has not been paid). [2 marks]

       E. How much does he need to pay at that time?   [1 mark]

    iv. A. An investor deposits a sum of 82000 dollars into a time deposit account today. The bank pays at a rate of 5 percent per annum. How much will it be in the next 5 years? If this amount compounds;
       (1) Quarterly   [1 mark]
       (2) Semi-annually   [1 mark]
       (3) Annually   [1 mark]
       (4) Monthly   [1 mark]

       B. A man on retirement on his 65th birthday receives a lump sum of 100000 dollars which he uses to purchase a pension which will be paid monthly in advance. Assuming a 4 percent interest and ignoring expenses and using approximation (where appropriate), obtain the monthly pension he receives.   [4 marks]

    v. If the normal rate of interest per annum is 6 percent when interest is compounded monthly;

       A. Find the annual equivalent rate of interest.   [1 mark]

B. Obtain the PV of 5000 dollars which is to be received in 5 years. [1 mark]

C. Obtain the amount of interest to be paid in arrears for the use of 1000 dollars over a one month period. [1 mark]

QUESTION TWO

(a) i. Explain the concept pure endowment factor and give its expected present value indicating who would wish to buy such a policy and explain why this benefit may be unpopular. [4 mark]

ii. For a return of 5000 dollars in 4.5 years' time, how much do you have to invest today at an effective interest rate of 5 percent per annum? [4 marks]

(b) i. Consider an n- year term endowment policy such that; it provides for a benefit either on the death of (x) or on survival of (x) to the end of the n-year term whichever event occurs first. [3 marks]
the death benefit is payable at the end of the month of death. [3 marks]
the death and survival benefits are both of 1 unit of money. [3 marks]
Let Z_1 and Z_2 be the present values of the death and survival benefits correspondingly, write these two random values in terms of $T(x)$, and hence find their probability mass functions.
Let $A^{(12)}{}_{x:\overline{n}|}$ be the expected present value of the benefits under this policy, so that

$$A^{(12)}{}_{x:\overline{n}|}=E(Z_1)+E(Z_2)$$

show that

$$A^{(12)}{}_{x:\overline{n}|}=1-d^{(12)}a^{(12)}_{x:\overline{n}|}$$

[3 marks]

QUESTION THREE

(a) i. Find the probability that a life who has reached 60 survives to his 65th birthday but dies within the 6 months. (Use the ELT 12 lifetable) [2 marks]

ii. The survival function
$$S(x) = e^x, \text{ for } x > 0.$$

Find the instantaneous death rate μ_x. If $K(x)$ is the curtate future life time of a life aged x, find $P[K(x) = k]$ and give the range. Find ${}_{30}q_{20}$. [4 marks]

iii. 1000 dollars is invested for three years. For the first 6 months the interest rate is 0.5 percent per month.
For the remaining period the APR is 5%. How much interest will have accrued by the end of the year. [3 marks]

(b) i. Suppose a sum of 10500 dollars is required in 5 years' from now. What will be the single sum of money that need to be deposited today in an account that pays 5% per annum compounded;

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(1) quarterly [1 mark]
(2) semi annually [1 mark]
(3) annually [1 mark]

ii. An investor wins a prize which can be taken either as a lump sum payment of 10000 dollars paid immediately or as monthly payments of 100 dollars per month paid in advance for life.
The investor is aged 60 and his mortality is given in the A1967 -70 Ultimate table.
Find the form of prize which is preferable and justify your answers. [5 marks]

iii. Explain the concepts
(1) Cash flow
(2) Annuity due
(3) Annuity certain [3 marks]

QUESTION FOUR

(a)   i. Consider placing a lump sum deposit of 8500 dollars today in a savings account that earns interest at 5% per annum. How long does it take to realize a savings balance of 15000 dollars, if the compounding period is
(1) quarterly
(2) annually
(3) semi annually
(4) monthly [4 marks]

ii. Suppose that a man actually survived to the age of 60. Find the surrender value of his policy at that time. [4 marks]

(b)   i. Consider an n- year temporary life annuity due with monthly payments at a rate of 1 p.a for a life age x now, that is , a level annuity-due, contingent on the survival of (x), payable monthly in advance at a rate of 1 p.a. for at most n years (the maximum number of payments possible is 12n) . Denote by $\ddot{a}_{x:\overline{n}|}^{(12)}$ its expected present value. Show that

$$\ddot{a}_{x:\overline{n}|}^{(12)} = \frac{1}{12} \sum_{j=0}^{12n-1} \frac{D_{x+\frac{1}{12}}}{D_x}$$

[4 marks]

ii. Barrack takes out a whole life insurance on his 50th birthday with a sum assured of 20000 dollars. Find the annual premium for this assurance if premiums are to start immediately and to be paid on each birthday. [4 marks]

iii. Suppose that for an initial investment of 1000 dollars you obtain a payment of 400 dollars after one year and 770 dollars after two years. Obtain the yield of this deal. [4 marks]

QUESTION FIVE

(a)   i. Use the life table ELT 12 to obtain the following results;
   (1) the probability that a newborn survives to age 21 [2 marks]
   (2) the expected number surviving to age 30 out of 1000 men age 20 [2 marks]
   (3) Let x and n be integers and let $0 < t < 1$. Let linear interpolation of the survival function between ages to show that

$$_{n|}q_x = t + {_{n|1}}q_x$$

[2 marks]

   ii. Suppose that Ababa actually died at the age of 60, so that the death benefit was paid on his 61st birthday. Calculate the actual loss made by the assurance company on John Byron's policy at the time when the benefit was paid. [4 marks]

(b)   i. Explain the following concepts
   (1) Temporary annuity
   (2) Immediate annuity inadvance
   (3) Pure endowment factor [3 marks]
   ii. Show that
$$(1) a_{x:\overline{n|}} = a_x - v^n {_n}p_x a_{x+n}$$

[1 mark]

$$(2) \sum_{t=1}^{n} v^{t-1} = \frac{1 - (1+i)^{-n}}{d}$$

[1 mark]

   iii. A man who is 38 years old wishes to purchase a life insurance policy which will pay his estate 50000 dollars at the end of the year of his death.If interest rate is 12%, find an expression for the actuarial present value for this benefit. [2 marks]

(c) An insurance company has to pay 20 million dollars 4 years from now to pensioners. Suppose that it can invest money in at an annual rate of 7% compounded semi annually. Obtain the effective annual rate and the amount the company should invest. [3 marks]