JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

SPECIAL RESIT 2 2020/2021 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SMA 202

COURSE TITLE: Vector Analysis

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (Compulsory) [30 Marks]

(a) Differentiate between a vector and a scalar, giving an example in each case. [4 Marks]

(b) The forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on an object O and the object moves. State the forces that are needed to prevent O from moving? [2 Marks]

(c) Show that addition of vectors is commutative. [3 Marks]

(d) Show that if $\mathbf{u}$ and $\mathbf{v}$ are non collinear, then $x\mathbf{u} + y\mathbf{v} = 0$ implies that $x = 0$ and $y = 0$.

[3 Marks]

(e) (i) Prove that $\nabla \cdot (\nabla \times \mathbf{A}) = 0$. [3 Marks]

(ii) If $\mathbf{A} = (xy^2, \sin y, z^2)$, find curl $\mathbf{A}$ at $(1, 3, -1)$. [3 Marks]

(f) The acceleration of any particle at any time t is given by

$$\mathbf{a} = 12\cos t\mathbf{i} - 8\sin 2t\mathbf{j} + 16t\mathbf{k}.$$

If the velocity $\mathbf{V}$ and displacement $\mathbf{r}$ are 0 at $t = 0$. Find $\mathbf{V}$ and $\mathbf{r}$ at any time.

[4 Marks]

(g) Evaluate the integral

$$\int_C [y\mathrm{d}x - x\mathrm{d}y]$$

using Green's theorem, where C is the circumference of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

[4 Marks]

(h) Use divergence theorem to find the flux output from the ellipsoid of volume V if $\mathbf{F} = 3x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. [4 Marks]

QUESTION TWO [20 Marks]

(a) Given the space curve $x = t$, $y = t^2$, $z = \frac{2}{3}t^3$. Find the:

(i) curvature, κ, and its radius, ρ. [8 Marks]

(ii) torsion, τ, and its radius, σ. [6 Marks]

(b) Find the directional derivative of $f(x, y, z) = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction of $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$. [6 Marks]

QUESTION THREE [20 Marks]

(a) If $\mathbf{A} = (3x^2 + 6y)\mathbf{i} - 14yz\mathbf{j} + 20xz^2\mathbf{k}$. Evaluate $\int_C \mathbf{A} \cdot \mathrm{d}\mathbf{R}$ from $(0,0,0)$ to $(1,1,1)$ along the following curves C.

(i) $x = t, y = t^2, z = t^3$. [3 Marks]

(ii) the straight lines from $(0,0,0)$ to $(1,0,0$ then to $(1,1,0)$ and finally to $(1,1,1)$. [4 Marks]

(iii) the straight line joining $(0,0,0)$ to $(1,1,1)$. [3 Marks]

(b) (i) State Green's theorem. [2 Marks]

(ii) Evaluate the integral along the curve C.

$$\int_C [(y - x^2 e^x)\mathrm{d}x + (\cos 2y^2 - x)\mathrm{d}y]$$

where C is the rectangle with vertices $(1,1), (0,1), (1,3)$ and $(0,3)$. [8 Marks]

QUESTION FOUR [20 Marks]

(a) Determine the constant c such that the vector $\mathbf{V} = (x + 3y)\mathbf{i} + (y - 2z)\mathbf{j} + (x + cz)\mathbf{k}$ is solenoidal. [5 Marks]

(b) (i) State Stoke's theorem. [2 Marks]

(ii) Verify Stoke's theorem for $A = (2x - y)\mathbf{i} - yz^2\mathbf{j} - y^2z\mathbf{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. [13 Marks]

QUESTION FIVE [20 Marks]

(a) Evaluate

$$\iint_S \mathbf{A} \cdot \mathbf{n}\mathrm{d}S$$

where $\mathbf{A} = 18z\mathbf{i} - 12\mathbf{j} + 3y\mathbf{k}$ and S is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant. [10 Marks]

(b) (i) State divergence theorem. [2 Marks]

(ii) Use divergence theorem to evaluate

$$\iint_S \mathbf{A} \cdot \mathbf{n}\mathrm{d}S$$

where $A = 4xz\mathbf{i} - y^2\mathbf{j} + yz\mathbf{k}$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. [8 Marks]