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SPECIAL RESIT EXAMINATION DECEMBER, 2020

SMA 208: INTRODUCTION TO ANALYSIS

INSTRUCTION: Attempt question ONE (**COMPULSORY**) and any other two questions only

QUESTION ONE (**COMPULSORY**) [30 marks]

(a) Define the terms: Complement of a set, Onto function and Equivalent sets. (6 marks)

(b) Given that $B = \{x : 0 \leq x < 1, x \in \mathbb{R}\}$, determine the infimum and the supremum and state whether they belong to B or not. (3 marks)

(b) Show that

$$\lim_{n\to\infty} \frac{3 + 2\sqrt{n}}{\sqrt{n}} = 2.$$

(6 marks)

(c) Let $x, y \in \mathbb{R}^+$. Show that there exists a positive integer t such that $tx > b$. (5 marks)

(d) State **without proof**, the Bolzano-Weierstrass theorem for sequences. (2 marks)

(e) Show that if P and Q are neighbourhoods of a point y then $P \cap Q$ is also a neighbourhood of y. (6 marks)

(f) Show that every open set is the union of open intervals. (4 marks)

(g) Given a sequence $\{a_n\}$, show that $\overline{\lim} a_n = +\infty$ if and only if $\{a_n\}$ is not bounded above. (3 marks)

QUESTION TWO [20 marks]

(a) Explain the following concepts: Closed set; Limit point; Convergent sequence; Point discontinuity and Asymptotic discontinuity. (10 marks)

(b) Show that every bounded sequence has a limit point. (10 marks)

QUESTION THREE [20 marks]

(a) Show that a set is open if its' complement is closed. (10 marks)

(b) Show that the set of rational numbers is not order complete. (10 marks)

QUESTION FOUR [20 marks]

(a) Show that every convergent sequence is bounded (10 marks)

(b) Given that $m, n \in \mathbb{R}$, show that $|m+n| \leq |m| + |n|$. (3 marks)

(c) Distinguish between Range of a function and Limit superior. (4 marks)

(d) Show that $f(y) = -y$ is 1-1 and onto for all $y \in \mathbb{N}$. (3 marks)

QUESTION FIVE [20 marks]

(a) If A is an open set and B is a closed set, show that $A \cup B^c$ is open. (5 marks)

(b) Show that the supremum of a nonempty set E (if it exists) is unique. (5 marks)

(c) Show that there is no rational number whose square is 7. (10 marks)