JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE(ACTUARIAL SCIENCE WITH IT)
SPECIAL RESIT 2 2020/2021 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SMA 312
COURSE TITLE: Operations Research I

EXAM VENUE: **STREAM: Bed Science Y3s2**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE(30 MARKS)

a) Define the following terms used in linear programming
   i) Linear inequality
   ii) Decision Variable
   iii) Slack variable
   iv) Objective function (8 marks)

b) State under what circumstances the following inequalities are used in operations research
   i) Less or equal to $(\leq)$
   ii) Greater or equal to $(\geq)$ (6 marks)

c) .Use Gauss Jordan method to solve the set of simultaneous equations

$$2x_1 + 3x_2 - x_3 = 12$$

$$x_1 - 2x_2 + 2x_3 = -11$$ (6 marks)

$$2x_1 + x_2 + 3x_3 = -8$$

d) A transporter has two types of trucks to transport sugar. Type A truck carries 2,000 bags while type B carries 3,000 bags per trip.The transporter has to transport 120,000 bags. He has to make not more than 50 trips. Type B trucks are to make atmost twice the number of trips made by type A.The transporter makes a profit of shs. 1,000 per trip for type A and shs. 2,0000 per trip for type B truck.
   (i) Formulate a linear programming problem hence state any three suitable methods that can be used in optimization. (7 marks)
   (ii) Form a dual of the primal problem in (i) above. (2 marks)

QUESTION TWO (20 MARKS)

a) A firm produces two types of perfume A and B. It can produce 8 bottles of perfume every minute. The quantity of perfume A must be less than thrice the quantity of perfume B while twice the quantity of perfume A must exceed that of perfume B. Additional information is as follows.

	Production cost per bottle(kshs)	Labourers per bottle	Profit per bottle
A	300	1	70
B	100	4	50
Available	900	20	

i) Develop a linear programming model based on the information above.
ii) Using graphical method and an isoprofit (search) line advise the firm on the number of bottles of each perfume to produce in order to maximize profit. (12 marks)

b) Use the Dual simplex method to solve the following LP problem

Minimize $Z = x_1 + 2x_2 + 3x_3$

Subject to

$$2x_1 - x_2 + x_3 \geq 4$$
$$x_1 + x_2 + 2x_3 \leq 8$$
$$x_2 - x_3 \geq 2$$
$$x_1, x_2, x_3 \geq 0$$

(8 marks)

QUESTION THREE(20 MARKS)

a) Three types of coupling units are produced by a firm. The times required for machining, polishing and assembling a unit of each type are included in the following table

	Type in hours per unit			
Type of unit	Machining	Polishing	Assembling	Profit per unit
A	2	4	1	30
B	4	2	1	40
C	3	1	2	50
Available time hours/week	160	96	80	

Advise the firm on how many of each type of coupling to produce in order to maximize profit. (14 marks)

b) Suppose in (a) above the available time in hours per week changed from 160 to 120 for machining, 96 to 112 for polishing and 80 to 60 for assembling. Would your original advise on levels of optimal production still stand. What would be the new optimal solution. (6 marks)

QUESTION FOUR (20 MARKS)

a) Use the big M simplex technique to solve the linear programming problem

Minimise $Z = -2x_1 + 8x_2$

Subject to

$$3x_1 + 4x_2 \le 120$$
$$-3x_1 + 4x_2 \ge 12$$
$$x_1 + 4x_2 \ge 60$$
$$x_1, x_2 \ge 0$$

(10 marks)

b) A company has three ware houses A, B and C and four stores W, X, Y and Z. The warehouses have altogether a surplus of 1600 units of a given commodity as follows

A	300
B	900
C	400

The four stores together need a total of 1600 units of the commodity as follows

W	400
X	600
Y	500
Z	100

The cost of transporting one unit in Ksh from each warehouse to store is shown in the table below

	W	X	Y	Z
A	400	200	600	400
B	500	700	900	200
C	350	820	530	680

Determine which method would be cheaper and by how much between Vogel's approximation and the least cost cell method as far as the cost of transport is concerned (10 marks)

QUESTION FIVE (20 MARKS)

a) Consider the linear programming problem

Maximise $Z = c_1x_1 + c_2x_2 + c_3x_3$

Subject to

$$\begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} x_1 + \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix} x_2 + \begin{pmatrix} a_{13} \\ a_{23} \end{pmatrix} x_3 + \begin{pmatrix} 1 \\ 0 \end{pmatrix} s_4 + \begin{pmatrix} 0 \\ 1 \end{pmatrix} s_5 = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$x_1, \ldots\ldots\ldots\ldots x_5 \geq 0$$

The optimal tableau is as follows

Basis	x_1	x_2	x_3	S_4	S_5	B
Z	3	0	0	0	4	20
X_3	1	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$
X_2	$\frac{1}{2}$	1	0	-1	2	2

i) Find the values of

$$\begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix}, \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix}, \begin{pmatrix} a_{13} \\ a_{23} \end{pmatrix} and \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

ii) Find the values of C_1 , C_2 and C_3 (9 marks)

iii) Find the range of values within which C_2 can fall without changing the optimal solution (3 marks)

b) Use two phase simplex method to to solve the LP problem

Maximize $Z = 22x_1 + 30x_2$

Subject to

$$3x_1 + 5x_2 \leq 130$$
$$-4x_1 + 5x_2 \geq 25$$
$$x_1 + 5x_2 \geq 75$$
$$x_1, x_2 \geq 0$$

(8 marks)