JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION 2012/2013

1ST YEAR 1ST SEMESTER EXAMINATION FOR THE DEGREE OF BSC. (ACTUARIAL SCIENCE)

(REGULAR)

COURSE CODE: SAC 103

TITLE: MATHEMATICAL MODELING

DATE: 29/4/2013   **TIME: 9.00-11.00AM**

DURATION: 2 HOURS

<u>INSTRUCTIONS</u>

1. **This paper contains SIX (6) questions**
2. **Answer question 1 (Compulsory) and ANY other 2 Questions**
3. **Write all answers in the booklet provided**

QUESTION ONE: Compulsory Question

(a) Find the derivative of $f(x) = \frac{1}{x^2}$ from first principles[6marks]

(b) Define the following terminologies

   (i)   Mathematical modeling[3marks]
   (ii)   A Model[3marks]
   (iii)   Mathematical model[3marks]

(c) A company producing computer components has established that on average, a new employee can assemble N(t) components per day after t days of on the job training, as given by N(t)= $\frac{40t}{t+2}$

   (i)   Find the rate of change of N(t) from 3 days to 6days[4marks]

   (ii)   Find the instantaneous rate of change of N(t) at 3days[4marks]

(d) Evaluate $\int_{-2}^{3}(6x^2 - 5)\,dx$[7marks]

   [Total 30marks]

QUESTION TWO

(a)A company determines that the cost= $C(x)$ of manufacturing x units of a commodity may be approximated by $c(x) = 100 + \frac{10}{x} + \frac{x^2}{200}$ How many units should be produced in order to minimize the cost?[10marks]

(b)Find all the solutions for the difference equations. You may leave your solutions in terms of factorials

(i) $a_n = n^4$[5marks]

(ii) $b_n = 2n^2 - n + 4$ **[5marks]**

QUESTION THREE

(a)(i)Find all the solutions of the difference equation $\Delta a_n = \gamma a_n$ where γ is some real number. What happens to the solutions when $\gamma = -1$ or 0[5marks]

(ii)Find the solutions to the difference equations $\Delta b_n = b_n + 1$[5marks]

(b)Coffee is priced in Kenya shillings at the opening of each market day at $p(x) = 5 + \frac{50}{x}$ where x is the amount in kgs of the commodity available on a given day. At what rate

will the price be changing when there are 7000kgs available and the amount of the commodity is decreasing at the rate of 400kg per day [10marks]

[Total 20marks]

QUESTION FOUR

(a) State and explain in a mathematical modeling (4marks)

(b) Find dy/dx if $y = \ln\left(x + \sqrt{x^2 + 1}\right)$

(4marks)

(c) The population growth $p(t)$ of a bacterial colony t hrs after observation begins is found to be changing at the rate $\frac{dp}{dt} = 200e^{0.1t} + 150e^{-0.03t}$ if the population was 200,000 bacteria when the observations began, what will the population be 12hrs later?

(6marks)

(d) The market research department of Kisumu's yattin supermarket has determined that for one store the marginal price $p'(x)$ at x tubes per week for Colgate toothpaste is given by $p'(x) = 0.015e^{-0.1x}$ find the price demand equation if the weekly demand is 50 tubes when the price of a tube is$2.35. find the weekly demand when the price of a tube is$1.89

(6marks)

[Total 20marks]

QUESTION FIVE

(a) The total sales(in thousands)for a home video games t months after the game is introduced is given by

$$s(t) = \frac{125t^2}{\sqrt{100 + t^2}}$$

(i) Find $s'(t)$ and simplify

(ii) Find $s(10)$ and $s'(10)$ and interpret results

(8marks)

(b) Determine all the stationary points of the function $f(x) = 3x^4 - 8x^2 - 3$ and differentiate between them

(6marks)

(c) Evaluate(i) $\int\left(\frac{2}{\sqrt[3]{x}} - 6\sqrt{x}\right)dx$

(ii)$\int_0^2 5x^2 (x^5 - 12)^9 dx$

(6marks)   **[Total 20marks]**