JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION ARTS, SPECIAL EDUCATION AND EDUCATION SCIENCE

1ST YEAR 2ND SEMESTER 2017/2018 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SMA 105

COURSE TITLE: INTRODUCTION PROBABILITY AND DISTRIBUTION THEORY

EXAM VENUE: **STREAM: (B.e.d ARTS, SPECIAL ed. & SCIENCE)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) In 20 independent trials, the probability of observing a certain outcome is 0.05 per trial. Find the probability of observing this trial outcome at least once. (3 Marks)

b) In a telephone sub network, the probability that a telephone is out of order per day is 0.0003. What is the probability of having 5 failures per day? (3 Marks)

c) The weekly demand for bread, in thousands of loaves from a local chain efficiency stores is a continuous random variable X having a probability density function given by

$$f(x)=\begin{cases}3(x-1) & 1<x<2\\ 0 & otherwise\end{cases}$$

Find the mean and variance of X. (5 Marks)

d) Show that the covariance of two random variables X and Y with means μ_X and μ_Y respectively is given by

$$\sigma_{XY} = E(XY)-\mu_X \mu_Y$$ (4 Marks)

e) Define the following terms as used in probability distributions. (6 Marks)

- Uniform random variable
- Exponential random variable
- Gamma random variable

f) Let X be a random variable with density function

$$f(x)=\begin{cases}x^2/3 & -1<x<2\\ 0 & elsewhere\end{cases}$$

Find the expected value of $g(x)=4x+3$ (3 Marks)

g) Compute the mean and variance of the following rectangular distribution (6 Marks)

$$f(x)=\begin{cases}\dfrac{1}{b-a} & a<x<b \quad -\infty<a<b<\infty\\ 0 & otherwise\end{cases}$$

QUESTION TWO (20 MARKS)

a) Suppose that the probability of a successful outcome in an experiment is given as 0.4. If 15 independent trials of the experiment are made, determine if;

i. $p(X=3)$

ii. $p(6\le X\le 9)$

iii. $p(X\ge 10)$

Given that X is the number of successes. (10 Marks)

b) The fraction X of male runners and the fraction Y of female runners who compete in marathon races are described by the joint density function ;

$$f(x,y)=\begin{cases}8xy & 0\le y\le x\le 1\\ 0 & elsewhere\end{cases}$$. Find the covariance of X and Y. (10 Marks)

QUESTION THREE (20 MARKS)

a) From a well –shuffled pack of 52 cards, 3 cards are taken at random. Find the probability of getting; (10 Marks)

i. at least 2 red cards .

ii. at best 2 red cards.

iii. no red cards.

b) Given a function of continuous random variable X as follows $f(x) = kx(1-x)$ with range space $R:\{X : 0 \le x \le 1\}$. Is $f(x)$ a density function? If so, find $P(A_1)$ where $A_1 = \{X : 0 \le x \le 1/3\}$ and $P(A_2)$ where $A_2 = \{X : x \ge 1/2\}$ (10 Marks)

QUESTION FOUR (20 MARKS)

Two regular tetrahedral are used in an experiment to obtain pairs of values (x_i, y_i) . The values on the face of the tetrahedral are numbered 1,2,3,4 and x_i is the value on the phase looking down on the tetrahedron A, y_i on the tetrahedron B or A, the larger of the two. Possible pairs (x_i, y_i) and corresponding probabilities are listed below

(X,Y)	(1,1)	(1,2)	(1,3)	(1,4)	(2,2)	(2,3)	(2,4)	(3,3)	(3,4)	(4,4)
$f(x,y)$	$1/16$	$1/16$	$1/16$	$1/16$	$2/16$	$1/16$	$1/16$	$3/16$	$1/16$	$4/16$

Obtain

i. F(2,3)

ii. $f_Y(y) \quad \forall y$

iii. $f_x(x) \quad \forall x$

QUESTION FIVE (20 MARKS)

a) A random variable X has a density function $f(x) = \begin{cases} \lambda e^{-\lambda x} & \lambda > 0; x \ge 0 \\ 0 & otherwise \end{cases}$. Find

i. $E(X)$

ii. $\text{var}(X)$ (10 Marks)

b) The probability mass function of a geometric distribution is given by

$$f(x) = \begin{cases} pq^x & x = 0,1,2..... \\ 0 & otherwise \end{cases}.$$

Compute the mean and variance of this geometric distribution. (10 Marks)