JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

RESIT 2

REGULAR (MAIN)

COURSE CODE: SMA 303

COURSE TITLE: COMPLEX ANALYSIS

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 marks)

a) Define the following terms as used in complex analysis

i) A complex number z. (2 marks)

ii) The principal argument. (2 marks)

iii) Interior point, z_o of a set S of the complex plane. (2 marks)

b) If $z_1 = 2 - 3i$ and $z_2 = 4 + 6i$, find $\frac{z_2}{z_1}$. (4marks)

c) Evaluate the complex function $f(z) = 2z^2 + 4\bar{z} - 4i$ at $z = 2 + 3i$. (3 marks)

d) Find an upper bound for the reciprocal of $z^5 - 6z + 2$ if $|z| = 2$. (5 marks)

e) Write the given complex number $z = -\sqrt{3} + i$ in polar form using

i)an argument $\theta \neq Arg(\theta)$ (3 marks)

ii) $\theta = Arg(\theta)$ (2 marks)

f) Compute the given complex limit, $\lim_{z \to i}(z^5 - z^2 + z)$. (3 marks)

g) Describe all the transformations represented by a complex mapping $f(z) = 4iz + 2 + 3i$. (4 marks)

QUESTION TWO (20 marks)

a) State De-Moivre's theorem, hence use it to evaluate $(2 - 2i)^5$, leaving your answer in the form $a + ib$; $a, b \epsilon \mathbb{R}$. (7marks)

b) Describe the set of points z in the complex plain that satisfy $|z| = |z - 2i|$. (5 marks)

c) Given $z_1 = \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ and $z_2 = \sqrt{3}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$, determine the value of $z_1 z_2$ (3 marks)

d) State the Cauchy's integral formula and hence evaluate

$\oint_C \frac{z^2 - 3z + 4i}{z + 2i} dz$; $|z| = 3$. (5 marks)

QUESTION THREE (20 marks)

a) Compute the n^{th} root for the complex number$\left(-1 - \sqrt{3}i\right)^{\frac{1}{4}}$. Hence sketch the roots w_0, w_1, w_2 and $w_3$on an appropriate circle centred at the origin. (6marks)

b) Use the quadratic formula to solve $z^2 + iz - 2 = 0$, hence or otherwise factorize the polynomial. (6marks)

c) Find the image of the line $y = 1$ under the complex mapping $w = z^2$ and represent the line and the mapping graphically. (5marks)

d) Find the value of the complex exponential form e^z at the point $z = 4 + \pi i$. (3marks)

QUESTION FOUR (20 marks)

a) Find solutions of the homogeneous differential equation $y^{II} + 2y^{I} + 2y = 0$. (5marks)

b) Find the derivatives of the following complex functions

i) $f(z) = 3z^4 - 5z^2 - 7z$, where $z \in \mathbb{C}$. (2marks)

ii) $f(z) = \frac{2z^3+4z}{4z+3}$, where $z \in \mathbb{C}$. (3marks)

c) Find the real and imaginary parts $u(x, y)$ and $v(x, y)$ of the complex function $f(z) = z^2 - 2z + 6$. (4marks)

d) Use L' Hospital's rule to compute $\lim_{z \to 2+i} \frac{z^2-4z+5}{z^3-z-10i}$. (6marks)

QUESTION FIVE (20 marks)

a) The function $f(z) = 3z^2 + 5z - 6i$ is analytic for all z. Determine whether the Cauchy-Riemann equations are satisfied or not. (5marks)

b) Evaluate $\int xydx + x^2dy$ over the C, where C is the graph of $y = x^3$ and $-1 \leq x \leq 2$. (6marks)

c) Given the function $u(x, y) = x^3 - 3xy^2$

i) Verify that $u(x, y)$ is harmonic in an appropriate domain D. (2marks)

ii) Find $v(x, y)$ the harmonic conjugate of $u(x, y)$. (5marks)

iii) Form the corresponding analytic function $f(z) = u + iv$. (2marks)