JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR THE DEGREE IN BED SCI/ ARTS AND BSC. ACTUARIAL SCIENCE

SPECIAL RESIT MAIN CAMPUS

COURSE CODE: SMA 304

COURSE TITLE: GROUP THEORY

EXAM VENUE: **STREAM: BSC. ACTUARIAL SCI AND BED SCI/ARTS**

DATE: **EXAM SESSION:**

TIME: 2 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**

Candidates must hand in their answer booklets to the invigilator while in the examination room.

QUESTION ONE (30 MARKS) COMPULSORY

a) Define: center of group, , group action, binary operation, (6 marks)

b) Show that the number of elements of symmetric group S_n is $n!$.

c) Let $\delta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 3 & 1 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 2 & 1 \end{pmatrix}$. (6 marks)

i) Find $\delta o \tau$

ii) Express τ is circle form

iii) Find δ^{-1}

d) Let Z_7^+ be a group of integers modulo 7 under addition. Find the order of 2,4,6. (5 marks)

e) Let $G=\{1,2,3\}$ be group under mulplication. Find the generator of the group. Is G cyclic? (5 marks)

f) State the Subgroup criterion theorem and Lagrange's Theorem. (4marks)

g) Prove that the identity element in the set A with binary operation is unique. (4 marks)

QUESTION TWO (20 MARKS)

a) Given the group (G,+) where $G=\{0,1,2,3\}$. Find the generator of the group G. Is G cyclic? (5 marks)

b) Construct a cayley table for the Z_5 of integer modulo 5 with respect to multiplication and show that Z_5 is indeed a group. (5 marks)

c) Given $\alpha\begin{pmatrix}1&2&3&4&5&6\\2&3&6&5&4&1\end{pmatrix}$ and $\beta\begin{pmatrix}1&2&3&4&5&6\\1&3&5&6&2&4\end{pmatrix}$. (5 marks)

Find $\alpha o \beta$, α^{-1}, $\beta^{-1}\alpha$

d) Prove that the center of a group is a normal subgroup of G. (5 marks)

QUESTION THREE (20 MARKS)

a) Consider the multiplication group $G=\{1,-i,-1,i\}$ and let $H=\{1,-1\}$ be its Subgroup. Find all the right coset and left cosetsof H in G. (3 marks)

b) Let $\phi: G \to H$ be a homomorphism of groups. Show that $Ker\phi$ and $\text{Im}\phi$ is a subgroup of G. (5 marks)

c) Describe the following terms (6 marks)
   i) Center of a group
   ii) Stabilier of a group
   iii) Kernel of homomorphism

d) Prove that if $\phi: G \to H$ is a homomorphism of groups and {e}is the identity of G then ϕ is monomorphism iff $Ker\phi=\{e\}$ (6 marks)

QUESTION FOUR (20 MARKS)

a) State and prove the fundamental theorem of group homomorphism (10 marks)

b) Let $a,b \in G$ and H be a subgroup of G. Show that the left cosets aH, bH have the same number of elements. (5 marks)

c) Let $G=\{Z_7 \setminus \{0\}\}$ be the group of integers modulo 7 under multiplication. Determine the generators of Z_7. Is Z_7 cyclic? (5 marks)

QUESTION FIVE (20 MARKS)

a) i) Show that if G is a group then, the inverse element $a^{-1} \in G$ is unique. (4marks)

ii) Let $G = \{0,1,2,3,4\}$ be a group of integers modulo5 under addition. Find the generators of the group G and hence determine whether G is cyclic. (6marks)

b) Draw the multiplication table for the set of $(\mathbb{Z}_5 \setminus \{0\}, \times)$ and determine whether it forms an abelian group (5marks)

c) i) Define homomorphism (2marks)

ii) Let $\phi: G \to H$ be a homomorphism. Show that if $a \in G,$ then $\phi(a^{-1}) = \phi(a)^{-1}$. (3marks)