JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION

ARTS, SPECIAL EDUCATION AND EDUCATION SCIENCE

RESIT 2

REGULAR (MAIN)

COURSE CODE: SMA 211

COURSE TITLE: PROBABILITY AND DISTRIBUTION THEORY II

EXAM VENUE: **STREAM: (B.e.d ARTS, SPECIAL ed. & SCIENCE)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) Outline FOUR advantages of sampling. (4 Marks)

b) Let X and Y be random variables with $\mu_X = 1, \mu_Y = 4, \sigma_X^2 = 1, \sigma_Y^2 = 1$ and $\rho_{XY} = 1/2$.
Find the mean and variance of $Z = 20X - 10Y$. (5 Marks)

c) Show that a random sample of size n from an infinite population that is $N(\mu, \sigma)$, has mean of sample $\mu_{\bar{x}} = \mu$ which is the population mean and standard error of the sample mean $\sigma_{\bar{x}} = \sigma / \sqrt{n}$ (5 Marks)

d) For a geometric distribution $p(x) = 2^{-x}$, $x = 1,2,3......$, prove that Chebyshev's inequality gives $p\{|X - 2| \leq 2\} > 1/2$. (6 Marks)

e) Let $\bar{x}$ denote the mean of a random sample of size 100 from a chi-square distribution with 50 degrees of freedom. Compute an approximate value of $p(49 < x < 51)$ (3 Marks)

f) Define the following terms as used in statistics
   i. Population
   ii. Sample
   iii. Sample error

g) Let $\bar{x}$ be the mean of a random sample of size 25 from a distribution that is normally distributed as $N(75,100)$. Find $p[71 < \bar{x} < 79]$ (4 Marks)

QUESTION TWO (20 MARKS)

a) Given that X is a continuous random sample, then X is said to have a chi-square distribution with probability density function given by

$$f(x) = \begin{cases} \frac{1}{\Gamma(n/2) 2^{n/2}} x^{n/2 - 1} e^{-x/2} & x > 0 \\ 0 & otherwise \end{cases}$$

i. Find the moment generating function of the chi-square distribution. (6 Marks)

ii. Find the mean and variance of the chi-square distribution. (4 Marks)

b) The probability distribution function if a random variable X is given below

$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

Show that, if k increases $p(|X - \mu|) \geq k\sigma$ decreases. (10 Marks)

QUESTION THREE (20 MARKS)

Let $X \sim N(0,1)$ be independent of another random variable Y which is a chi-square with r degrees of freedom. Consider a new variable $t = \frac{X}{\sqrt{Y/r}}$, where $-\infty < X < \infty$ and $0 < Y < \infty$.

Find the probability distribution of t.

QUESTION FOUR (20 MARKS)

Let $X_1, X_2, \ldots\ldots X_n$ be random variables such that X_i's are chi- square with r_i degrees of freedom where $i = 1,2,3.......,n$. Let each X_i and X_j be independent.

a) Obtain the joint probability distribution function of X_1 and X_2

b) Obtain the probability distribution function of $f = \frac{X_1/r_1}{X_2/r_2}$

QUESTION FIVE (20 MARKS)

Suppose that X and Y are jointly distributed random variables with probability distribution function given by

$$f(X,Y) = \begin{cases} \frac{1}{8}(X+Y) & 0 < X < 2 \quad 0 < Y < 2 \\ 0 & otherwise \end{cases}$$

Compute the coefficient of correlation between X and Y