JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION ARTS, SPECIAL EDUCATION AND EDUCATION SCIENCE

RESIT 2

REGULAR (MAIN)

COURSE CODE: SMA 210

COURSE TITLE: PROBABILITY AND DISTRIBUTION THEORY I

EXAM VENUE: **STREAM: (B.e.d ARTS, SPECIAL ed. & B.ed SCIENCE)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) Let X and Y have a bivariate probability density function given by

$$f(x,y)=\begin{cases} \frac{3}{2}x^2 & 0\le x\le 2; 0\le y\le 1 \\ 0 & otherwise \end{cases}$$

Obtain marginal densities of X and Y. (4 Marks)

b) Suppose that the joint probability distribution function of X and Y is

$$f(x,y)=\begin{cases} \frac{3}{16}(4-2x-y) & x>0; y>0; 2x+y<4 \\ 0 & otherwise \end{cases}$$

Determine:

i. The conditional probability density function of Y given X. (4 Marks)

ii. Compute $\Pr[Y\ge 2/X=0.5]$ (4 Marks)

c) Outline TWO properties of covariance of two random variables. (2 Marks)

d) Suppose that X and Y are random variables of $\text{var}(X)=9$, $\text{var}(Y)=4$ and $\rho_{XY}=-\frac{1}{6}$.

Determine:

i. $\text{var}(X+Y)$ (2 Marks)

ii. $\text{var}(X-3Y+4)$ (2 Marks)

e) Given that X_1 and X_2 are random variables with joint probability distribution function given by

$$f(x_1,x_2)=\begin{cases} x_1+x_2 & 0<x_1<1, 0<x_2<1 \\ 0 & otherwise \end{cases}$$

Determine whether or not X_1 and X_2 are independent. (5 Marks)

f) Consider a two dimensional random variable (X_1, X_2) having a density function given by

$$f(x_1,x_2)=\begin{cases} 8x_1x_2 & 0\le x_1\le 1; 0\le x_2\le 1 \\ 0 & otherwise \end{cases}$$

Compute:

i. $E(3X_1+2X_2)$ (4 Marks)

ii. $E(X_1X_2)$ (3 marks)

QUESTION TWO (20 MARKS)

a) Suppose that X is a random variable such that $0<\delta_X^2<\infty$ and that $Y=aX+b$ for some constant a and b where $a\ne 0$. Show that if $a>0$ then $\rho_{XY}=1$ and if $a<0$ then $\rho_{XY}=-1$ (4 Marks)

b) Describe the regression between X and Y from a joint probability distribution function given by

$$f(x,y)=\begin{cases}\frac{1}{2}xy & 0<y<x:0<x<2\\ 0 & otherwise\end{cases}$$ (16 Marks)

QUESTION THREE (20 MARKS)

a) Show that the moment generating function of a bivariate normal distribution is given by
$m(t_1,t_2)=\exp\left\{t_1\mu_x+t_2\mu_y+\frac{1}{2}\left[t_1^2\delta_x^2+2\rho t_1t_2\delta_x\delta_y+t_2^2\delta_y^2\right]\right\}$ (10 Marks)

b) Show that if X and Y are random variables with a bivariate normal distribution, then
$E(X)=\mu_x$, $E(Y)=\mu_y$, $\text{var}(X)=\delta_x^2$, $\text{var}(Y)=\delta_y^2$ and $\text{cov}(XY)=\rho\delta_x\delta_y$ (10 Marks)

QUESTION FOUR (20 MARKS)

a) Consider two independent random variables X_1 and X_2 both coming from a population with probability density function

$$f(x)=\begin{cases}1 & 0<x<1\\ 0 & otherwise\end{cases}$$

Suppose we define two other random variables $Y_1=X_1+X_2$ and $Y_2=X_1-X_2$. Obtain;

i. the joint probability distribution of Y_1 and Y_2

ii. the marginal probability distribution function Y_1 (10 Marks)

b) Define a Beta distribution. (2 Marks)

c) Obtain the mean and variance of a Beta distribution. (8 Marks)

QUESTION FIVE (20 MARKS)

Suppose that X_1 and X_2 are jointly distributed random variables with probability distribution function given by

$$f(x_1,x_2)=\begin{cases}\frac{1}{8}(x_1+x_2) & 0\le x_1\le 2;0\le x_2\le 2\\ 0 & otherwise\end{cases}$$

Compute the coefficient of correlation between X_1 and X_2