JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

RESIT 2

MAIN REGULAR

COURSE CODE: SMA 200

COURSE TITLE: CALCULUS II

EXAM VENUE: **STREAM:**

DATE: EXAM SESSION:

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY) (30 marks)

a) Evaluate the following definite integral $\int_1^2 \left(3x^5 - \frac{1}{2}x^4 + 7x^2 + x - 3\right)$ (4 marks)

b) Using $\int_0^8 \frac{1}{4}xdx$, show that the idea behind integration is that we can effectively compute many quantities by breaking them into small pieces, and then summing the contributions from each small part (4 marks)

c) By multiplying the integrand by appropriate form of 1, evaluate the following integral

$\int \frac{1}{1-\sec\theta} d\theta$ (4 marks)

d) Using trigonometric substitution, evaluate the indefinite integral

$\int \frac{\sqrt{4-x^2}}{x^2} dx$ (5 marks)

e) Evaluate the given integral using appropriate substitution:

$\int \frac{14x}{\sqrt{x^2-9}} dx$ (4 marks)

f) Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 4$ (4 marks)

g) Evaluate appropriately $\int_0^1 e^x dx$ using Simpson's rule with eleven ordinates. (5 marks)

QUESTION TWO (20 marks)

a) Using appropriate substitution to reduce to standard form, evaluate the integral

$\int \frac{3}{\sqrt{x}\left(\sqrt{x}-1\right)} dx$ (5 marks)

b) By first separating the fraction, evaluate the following integral

$\int \frac{2x-3}{x^2+4} dx$ (5 marks)

c) Applying the technique of linear substitution, evaluate

$\int \frac{e^{2x}}{e^x - 3} dx$ (5 marks)

d) Verify by differentiation that the formula is correct:

$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\frac{x}{a} + c$ (5 marks)

QUESTION THREE (20 marks)

a) Use partial fraction method to evaluate the integral:

$$\int_1^2 \frac{4y^2 - 7y - 12}{y^3 - y^2 - 6y}\,dy \quad \text{(7 marks)}$$

b) Evaluate the following integral using appropriate technique:

$$\int e^{2x} \cos 3x dx \quad \text{(6 marks)}$$

c) Use reduction formula to evaluate the integral below:

$$\int 2\sec^3 \pi x dx \quad \text{(7 marks)}$$

QUESTION FOUR (20 marks)

a) The rate of change in the mass M (in grams) with respect to the total length L (in millimetres) of a given type of fish can be modeled by

$$\frac{dM}{dL} = 0.000116L^{1.9004}$$

A fish that is 200 millimetres long has a mass of 100grams.

(i) Find a model for the mass function. (4 marks)

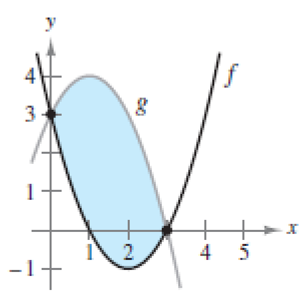
(ii) Find the mass of a fish that is 250 millimetres long. (4 marks)

(iii) Find the net change in the mass of a fish when the length increases from 200 to 300 millimetres. (4 marks)

b) Find the total area of the shaded region shown below:

$f(x) = x^2 - 4x + 3$

$g(x) = -x^2 + 2x + 3$



(8 marks)

QUESTION FIVE (20 marks)

a) Evaluate $\int_0^4 \sqrt{4 - x^2}\,dx$ by trapezium rule with eight subintervals (5 marks)

b) Find a power series for the logarithmic function

$L(x) = \ln(1 + x)$ (5 marks)

c) Show that the Taylor series about $x = 0$ for the function $f(x) = e^x$ is $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ (5 marks)

d) Evaluate the following improper integral

$$\int_0^9 \frac{1}{\sqrt{9 - x}}\,dx \quad \text{(5 marks)}$$