JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION

ARTS, SPECIAL EDUCATION AND EDUCATION SCIENCE

RESIT 2

REGULAR (MAIN)

COURSE CODE: SMA 105

COURSE TITLE: INTRODUCTION PROBABILITY AND DISTRIBUTION THEORY

EXAM VENUE: **STREAM: (B.e.d ARTS, SPECIAL ed. & SCIENCE)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) In 20 independent trials, the probability of observing a certain outcome is 0.05 per trial. Find the probability of observing this trial outcome at least once. (3 Marks)

b) In a telephone sub network, the probability that a telephone is out of order per day is 0.0003. What is the probability of having 5 failures per day? (3 Marks)

c) The weekly demand for bread, in thousands of loaves from a local chain efficiency stores is a continuous random variable X having a probability density function given by

$$f(x) = \begin{cases} 3(x-1) & 1 < x < 2 \\ 0 & otherwise \end{cases}$$

Find the mean and variance of X. (5 Marks)

d) Show that the covariance of two random variables X and Y with means μ_X and μ_Y respectively is given by

$$\sigma_{XY} = E(XY) - \mu_X \mu_Y$$ (4 Marks)

e) Define the following terms as used in probability distributions. (6 Marks)

- Uniform random variable
- Exponential random variable
- Gamma random variable

f) Let X be a random variable with density function

$$f(x) = \begin{cases} x^2/3 & -1 < x < 2 \\ 0 & elsewhere \end{cases}$$

Find the expected value of $g(x) = 4x + 3$ (3 Marks)

g) Compute the mean and variance of the following rectangular distribution (6 Marks)

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a < x < b \quad -\infty < a < b < \infty \\ 0 & otherwise \end{cases}$$

QUESTION TWO (20 MARKS)

a) Suppose that the probability of a successful outcome in an experiment is given as 0.4. If 15 independent trials of the experiment are made, determine if;

i. $p(X = 3)$

ii. $p(6 \leq X \leq 9)$

iii. $p(X \geq 10)$

Given that X is the number of successes. (10 Marks)

b) The fraction X of male runners and the fraction Y of female runners who compete in marathon races are described by the joint density function ;

$f(x, y) = \begin{cases} 8xy & 0 \leq y \leq x \leq 1 \\ 0 & elsewhere \end{cases}$. Find the covariance of X and Y. (10 Marks)

QUESTION THREE (20 MARKS)

a) From a well –shuffled pack of 52 cards, 3 cards are taken at random. Find the probability of getting; (10 Marks)

i. at least 2 red cards .
ii. at best 2 red cards.
iii. no red cards.

b) Given a function of continuous random variable X as follows $f(x) = kx(1-x)$ with range space $R:\{X : 0 \le x \le 1\}$. Is $f(x)$ a density function? If so, find $P(A_1)$ where $A_1 = \{X : 0 \le x \le 1/3\}$ and $P(A_2)$ where $A_2 = \{X : x \ge 1/2\}$ (10 Marks)

QUESTION FOUR (20 MARKS)

Two regular tetrahedral are used in an experiment to obtain pairs of values (x_i, y_i). The values on the face of the tetrahedral are numbered 1,2,3,4 and x_i is the value on the phase looking down on the tetrahedron A, y_i on the tetrahedron B or A, the larger of the two.

Possible pairs (x_i, y_i) and corresponding probabilities are listed below

(X,Y)	(1,1)	(1,2)	(1,3)	(1,4)	(2,2)	(2,3)	(2,4)	(3,3)	(3,4)	(4,4)
$f(x,y)$	$1/16$	$1/16$	$1/16$	$1/16$	$2/16$	$1/16$	$1/16$	$3/16$	$1/16$	$4/16$

Obtain

i. F(2,3)
ii. $f_Y(y) \quad \forall y$
iii. $f_x(x) \quad \forall x$

QUESTION FIVE (20 MARKS)

a) A random variable X has a density function $f(x) = \begin{cases} \lambda e^{-\lambda x} & \lambda > 0; x \ge 0 \\ 0 & otherwise \end{cases}$. Find

i. $E(X)$
ii. $\operatorname{var}(X)$ (10 Marks)

b) The probability mass function of a geometric distribution is given by

$$f(x) = \begin{cases} pq^x & x = 0,1,2..... \\ 0 & otherwise \end{cases}.$$

Compute the mean and variance of this geometric distribution. (10 Marks)