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(Special Resit)

## SMA 101: ANALYTIC GEOMETRY

INSTRUCTION: Answer Question ONE and ANY other TWO questions.

### QUESTION ONE (COMPULSORY) – 30 MARKS

- a) Use the third order matrix determinant to determine the equation of a line passing through the points A(-2, 1) and B(4, -5) giving your answer in double intercept form. (4 marks)
- b) A point C has coordinates C(6.5,0), determine the angle ACB taking A and B from part a) above. (4 marks)
- c) Given two parallel lines  $4y - 3x = -3$  and  $4y = 3x - 28$ , determine the distance between them and the equation of a line that passes midway between them. (4 marks)
- d) Determine the equation of a circle which passes through (-6, 5), (-2, 7) and (6, 1) giving your answer in the form  $Ax^2 + By^2 + Cx + Dy + E = 0$ . Give the equations of four lines that intersect to form the square that circumscribes the circle. (10 marks)
- e) Give polar coordinates in two forms for each of the following points, one with positive polar coordinates and another with negative polar coordinates.
- i) (-4, 5)                      ii) (-2, -7) (4 marks)
- f) Sketch the curve  $r = \frac{6}{2 + 6\cos\theta}$  stating and clearly indicating the vertex/vertices and the focus/foci (4 marks)

**QUESTION 2(20 MARKS)**

a) The equation of an ellipse is given by  $36x^2 + 25y^2 - 216x + 200y - 176 = 0$

Find on the  $xy$  plane

(i) The centre of the ellipse (4 marks)

(ii) The coordinates of the vertices (2 marks)

(iii) The foci (2 marks)

(iv) The eccentricity (1 mark)

(vi) The directrices (2 marks)

(vii) The area of the ellipse (3 marks)

b) A second degree curve is represented by the equation  $x^2 - 2xy + y^2 - 16x - 48y = 0$ . By eliminating the cross product term identify the conic section hence give its equation on the  $x'y'$  plane and state the equation of the axis. (6 marks)

**QUESTION 3(20 MARKS)**

a) The equation of a hyperbola is given as  $9x^2 - 36x - 4y^2 - 24y - 36 = 0$ . Find

(i) The coordinate of the centre. (4 marks)

(ii) The foci of the hyperbola on the  $xy$  plane. (2 marks)

(iii) The vertices on the  $xy$  plane. (2 marks)

(iv) The asymptotes on the  $x'y'$  plane and on the  $xy$  plane. (4 marks)

(vi) The eccentricity (1 mark)

(vii) The directrices on the  $x'y'$  plane and on the  $xy$  plane. (3 marks)

b) Give the cartesian equation of the following pairs of parametric equations

(i)  $x = t - t^2, y = t^2 + t^3$  (2 marks)

(ii)  $x = \frac{2t}{1+t^2}, y = \frac{2t^2}{1+t^3}$  (2 marks)

**QUESTION 4(20 MARKS)**

a) Find the parametric equation of the following Cartesian equations

(i)  $x^3 + y^3 = 3xy$  (2 marks)

(ii)  $xy = x - y$  (2 marks)

(iii)  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  (2 marks)

b) (i) A conic section has the equation  $3x^2 + 4\sqrt{3}xy - y^2 = 7$ . Rotate the axes of the conic section by eliminating the cross product term. (8 marks)

(ii) Identify the conic section on the new  $x'y'$  plane (2 marks)

(iii) Give the coordinates of the focus/foci of the conic on the  $x'y'$  plane. (2 marks)

(iv) Find the eccentricity of the conic section (2 marks)

**QUESTION 5(20 MARKS)**

a) Sketch and give the name of the polar curves  $r = 1 + 3 \cos \theta$  (6 marks)

b) Identify the conic sections given below

$i) x^2 - 2xy + y^2 - 16x - 48 = 0$	$iii) r = \frac{4}{2 - 2 \cos \theta}$	(6 marks)
$ii) 3x^2 - 4\sqrt{3}xy - y^2 = 24$	$iv) r(4 + 3 \sin \theta) = 0$	

c) A parabola has the y- intercepts -1 and 2

Find (8 marks)

- (i) The equation of the parabola
- (ii) The equation of the axis of the parabola
- (iii) The vertex and focus of the parabola
- (iv) The equation of the directrix of the parabola