JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE

RESIT 2

MAIN CAMPUS

COURSE CODE: SMA 100

COURSE TITLE: BASIC MATHEMATICS

EXAM VENUE: **STREAM: EDUCATION, ACTUARIAL**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

<u>QUESTION ONE</u> (30 marks)

a) Define the following terms as used in set theory and give example in each case.
   i) Cardinality of a set (2marks)
   ii) Empty set (2marks)

b) Given that $X = \{a, b, c\}$ and $Y = \{1, 0\}$, show that the $X \times Y \neq Y \times X$ (2marks)

c) i) Use Binomial theory to determine the expansion of $(3x - 2y)^5$ (4marks)

   ii) A defensive football squad consists of 25 players. Of these, 10 are linemen, 10 are linebackers and 5 are safeties. How many different teams of 5 linemen, 3 linebackers and 3 safeties can be formed? (3marks)

d) Solve the equation: $\ln x = \ln(x + 6) - \ln(x - 4)$ (3marks)

e) Prove the identity

$$\frac{\sin\theta}{1+cos\theta} + \frac{1+cos\theta}{sin\theta} = 2csc\theta$$ (5marks)

f) An oil company bores a hole 40 metres deep. Estimate the cost of boring if the cost is \$ 15 for the first metre with an increase in cost of \$ 2 per metre for each succeeding metre. (4marks)

g) Write $(1 + i)^5$ in the standard form $a + bi$ (5marks)

<u>QUESTION TWO</u> (20 marks)

a) Find the power set of $G = \{x: x \in \mathbb{N}\ and\ x^2 - 4x + 3 = 0\}$ (4marks)

b) Let $U = \{1, \dots, 9\}$ be the universal set and $A = \{1 \leq x \leq 4\}$, $B = \{2,4,6,8\}$ and $C = \{2 < x < 7\}$. Find;

   i) $(A \cap C)^c$ (2marks)

   ii) $(A \cup B)^c$ (2marks)

   iii) $B \backslash C$ (1mark)

   iv) $A^c \cap (B^c \cap C)$ (3marks)

c) Draw the Venn diagram and shade the region corresponding to $(P^c \cap Q) \cap R^c$ (3marks)

d) Prove the following De Morgan's law of set operations:

$$(A \cup B)^c = A^c \cap B^c$$ (5marks)

<u>QUESTION THREE</u> (20 marks)

a) Use Cramer's rule to solve the following systems of linear equations
x - 3z = -2
3x + y - 2z = 5
2x + 2y + z = 4 (10marks)

b) In a group of 191 students, 10 are taking English, computer science and music, 36 are taking English and computer science, 20 are taking English and music, 18 are taking computer science and music, 65 are taking English, 76 are taking computer science and 63 are taking music. Find how many students are taking
i) English and music but not computer science
ii) Computer science and music but not English
iii) Computer science and neither English nor music
iv) None of the three subjects

(10marks)

<u>QUESTION FOUR</u> (20 marks)

a) Solve: $5^{x-2} = 3^{3x+2}$ (4marks)

b) Let f and g be the functions from the set of integers to the set of integers defined by $g(x) = 2x^2$ and $h(x) = x + 2$. Find
i) $(goh)(x)$ (2marks)
ii) $(hog)(x)$ (2marks)
iii) $(gogoh)(x)$ (2marks)

c) Find the inverse of $f(x) = \frac{4}{2-x}$ where $x \neq 2$ (3marks)

d) Solve the equation $2sin^2\theta = cos\theta + 1$ for θ in the range $0^o \leq \theta \leq 360^o$ (4marks)

e) Use the remainder theorem to evaluate whether $x = 2$ is a solution of

$f(x) = 6x^4 - x^2$-3x + 2 (3marks)

<u>QUESTION FIVE</u> (20 marks)

a) An arithmetic progression has first term $\log_2 27$ and a common difference $\log_2 x$.

i) Show that the fourth term can be written as $\log_2 27x^3$ (3marks)
ii) Given that the fourth term is 6, find the exact value of x (2marks)

b) A geometric progression has first term $\log_2 27$ and a common ratio $\log_2 y$.
i)Find the set of values of y which the geometric has a sum to infinity (2marks)
ii) Find the exact value of y for which the sum to infinity of the geometric progression is 3 (5marks)

c) The first 3 terms of a sequence are $2x$, $x + 4$ and $2x - 7$ respectively.
i)Verify that when $x = 8$ the terms forma geometric progression and find the sum to infinity in this case. (4marks)
ii) Find the other possible value of x that also gives a geometric progression. (4marks)