JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY SPECIAL/RESITS EXAMINATION FOR BSc/BEd IN MATHEMATICS

SPECIAL RESIT 2020 /2021ACADEMIC YEAR

MAIN CAMPUS- SPECIAL RESIT

COURSE CODE: SMA201

COURSE TITLE: LINEAR ALGEBRA II

EXAM VENUE: **STREAM: BSc BSc/BEd**

TIME: 2 HOURS **EXAM SESSION:**

Instructions:

Answer question1 and **any other two** questions
Show all the necessary working

1.Candidates are advised not to write on the question paper
2.Candidates must hand in their answer booklets to the invigilator while in the examination room

Question1 [30marks] Compulsory

(a) If $A=\begin{bmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix}, B=\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 1 \end{bmatrix}, C=\begin{bmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 0 & 0 & -1 \end{bmatrix}$

(i)Find the matrix products BC , CB and hence factorize **A** *into the* form $A=LU$

(ii) State the use matrix factorization in solving the system $A\underset{\sim}{X}=\underset{\sim}{b}$. Do not solve the system.

[8marks]

(b) Let $P=\begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}, v=\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $u=\begin{bmatrix} 2 \\ 10 \end{bmatrix}$ be real matrices.

(i)Compute the images Pv , Pu

(ii) On same diagram sketch the general displacement vectors Pv , Pu , v , u passing through the origin. State which of the vectors v , u is an eigenvector of P. [7marks]

(c) Let U be a vector space over field F of complex numbers.

Suppose $a,b,c,d \in U;\ k,l \in F$.

(i) Define a rule $*$, on U together with F for which $*$ is known to be an inner product on U.

(ii) Show that the rule $\oplus$, defined on the R^2 vector space by :

$\begin{pmatrix} x \\ y \end{pmatrix} \oplus \begin{pmatrix} u \\ v \end{pmatrix} = 200xu + 200yv$ is an inner product. **[7marks]**

(d) (i)Prove that the set 4by4 matrices $W=\left\{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -14 \\ 0 & 4 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 14 \\ 0 & 0 & 0 \\ -4 & 0 & 0 \end{bmatrix}\right\}$ is linearly independent.

(ii) Determine whether the matrix $N=5\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & -4 \\ 0 & 2 & 0 \end{bmatrix}$ is in the $span\{W\}$ [8marks]

Question2 [20marks]

Define $T:P_2 \to R^3$ **by** $T\left(p\left(t\right)\right)=\begin{bmatrix} p\left(-10\right) \\ p\left(0\right) \\ p\left(10\right) \end{bmatrix}; p\left(t\right)=3at^2+bt+c$

(a)Find the images under T of $p\left(t\right)=t^2+9t+4$, $p\left(t\right)=-9t+4$ and $p\left(t\right)=4$

(b)(i) Show that T is a linear transformation.

(ii) Show that T is a linear operator.

(c) Find the matrix of T relative to the basis $\left\{1,\ t\ ,t^2\right\}$ for P_2 and the standard basis for R^3

Question3 [20 marks]

Consider the vector space of R^4 with the inner product $\langle\ ,\ \rangle$:

$\left\langle \underline{x},\underline{y}\right\rangle=5x_1y_1+5x_2y_2+x_3y_3+x_4y_4$; $\underline{x}=\left[x_1,x_2,x_3,x_4\right]$, $\underline{y}=\left[y_1,y_2,y_3,y_4\right]$, $x_i,y_i\in R$; $\underline{x}$, $\underline{y}\in R^4$

(a)Show that $\left\langle \underline{x},\underline{x}\right\rangle\geq 0$ **[6 marks]**

(b) Show that $\left\langle \underline{x},\underline{y}\right\rangle=\left\langle \underline{y},\underline{x}\right\rangle$ **[5 marks]**

(c)Apply the Gram-Schmidt process to the set of linearly independent vectors

$\left\{v_1=\left[-1,-1,1,1\right],v_2=\left[-1,-1,-1,-1\right],v_3=\left[-1,-1,-1,1\right]\right\}$

to obtain an orthonormal basis $\left\{w_1,w_2,w_3\right\}$ **.**

[9 marks]

Question4 [20marks]

(a) State and prove Caley-Hamilton theorem. **[4marks]**

(b)Let $A=\begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$

(i) Find the characteristic equation of A. **[6marks]**

(ii) Compute the matrices A^2, A^3 **[6marks]**

(ii) Prove that $4I - A^3\text{-}3A^2 = 0_{3\times3}$ **[4marks]**

Question5 [20marks]

Given $A = \begin{pmatrix} 8 & -2 & -3 & 1 \\ 7 & -1 & -3 & 1 \\ 6 & -2 & -1 & 1 \\ 5 & -2 & -3 & 4 \end{pmatrix}$ is matrix of linear operator T

(a) If $v_1 = [1, 1, 1, 1]^t$, $v_2 = [70000, 70000, 70000, 0]^t$, $v_3 = [40, 100, 40, 40]^t$,

$v_4 = [300, 300, 500, 300]^t$, evaluate Av_1, Av_2, Av_3, Av_4.[4 marks]

(b) Find $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, the eigenvalues of T and set $U = \{u_1, u_2, u_3, u_4\}$ the corresponding eigenvectors [16marks]

SMA 201: LINEAR ALGEBRA II **(42HRS)**
Credit Hours: 3

Pre-requisites: Linear Algebra I

Purpose

To apply basic algebra concepts in understand Matrix analyses and Eigen value theorems and to extend linear algebra I to vector spaces and concepts of diagonalization and decomposition

Expected Learning Outcomes

By the end of the course the learner should be able to:

i) Evaluate determinant
ii) Represent a function by matrix
iii) Use Eigen value and Eigen vectors to determine whether a given matrix is diagonalizable
iv) Apply inner product theory geometry

Course Content

Field axioms. Vector spaces over an arbitrary field. Linear mapping and their matrices with respect to an arbitrary basis, the change of basis. Conjugation of eigenvectors theorem. Invariant subspaces. Quadratic forms. inner product spaces

Teaching / Learning Methodologies: Lectures; Practical work (Minitab Package); Class discussion

Instructional Materials and Equipment: Overhead Projectors; Power Point; Flip Charts; Handouts; Chalk board

Course Assessment: Examination - 70%; Continuous Assessments (Exercises and Tests) - 30%; Total - 100%