JARAMOGI OGINGA ODINGA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FIRST YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF ……………

SMA *3112*: MATHEMATICS II

Date: …….. April, 2013 ***Time: ……. - ………***

INSTRUCTIONS:

1. This examination paper contains five questions. Answer **question one,** and **any other two** questions.
2. Start each question on a fresh page.
3. Indicate question number clearly at the top of each page.

QUESTION ONE (30 marks)

a) Find the equation of the straight line through $(-1,-3)$

   i. Parallel to line $4x+3y-5=0$, (3 marks)

   ii. Perpendicular to line $5x-2y-1=0$. (3 marks)

b) Use the following matrices

$$A=\begin{bmatrix}0 & 3 & 5\\ 1 & 2 & 6\end{bmatrix},\ B=\begin{bmatrix}4 & 1 & 0\\ -3 & 3 & -2\end{bmatrix}$$

to evaluate the given expression

$2A-3B$ (4 marks)

c) Determine the point of discontinuity (if any) of the function $f(x)$

$$f(x)=\frac{x^2-5x+4}{x-4}$$

If the continuity is removable, define the function to make it continuous. (5 marks)

d) Find:

   i. $\lim\limits_{x\to 4}\frac{\sqrt{x}-2}{x-4}$ (3 marks)

   ii. $\lim\limits_{x\to +\infty}\frac{2x+5}{x^2-7x+3}$ (3 marks)

e) Find the derivative of the function $f(x)=\frac{1+x-4\sqrt{x}}{x}$. (4 marks)

f) Evaluate the integral $\int x^3\left(1+9x^4\right)^{\frac{-3}{2}}\,dx$ (5 marks)

QUESTION TWO (20 marks)

a) The coordinates of the vertices A,B,C of the triangle ABC are $(-3,7)$, $(2,19)$, $(10,7)$ respectively. Prove that the triangle is isosceles. (6 marks)

b) The points A, B and C have coordinates $(8,1)$, $(4,-2)$ and $(-2,4)$ respectively. Find the coordinates of D, E and F, the mid-points of BC, CA and AB respectively. Find the equations of the lines AD, BE, and the coordinates of G, their point intersection. Prove that C, G, F are in a straight line. (14 marks)

QUESTION THREE (20 marks)

a) Evaluate the matrix product:

$$\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 1 & 0 & -1\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & -1\\ 1 & 0\end{bmatrix}$$ (5 marks)

b) Solve for x:

$$\begin{vmatrix}x & 1 & 2\\ 1 & x & 3\\ 0 & 1 & 2\end{vmatrix}=-4x.$$ (5 marks)

c) Solve the system of equations below using Cramer's Rule if it is applicable. If Cramer's rule is not applicable say so:

$$\begin{cases} 2x + y - z = 3 \\ -x + 2y + 4z = -3 \\ x - 2y - 3z = 4 \end{cases}$$ (10 marks)

QUESTION FOUR (20 marks)

a) Evaluate the integral by using a substitution to reduce it to standard form:

$\int x^5 e^{1-x^6} dx$ (5 marks)

b) Find the derivative of y with respect to x:

$y = \dfrac{\ln \sqrt[3]{x^2}}{x^4}$ (5 marks)

c) Evaluate the following integral:

$\int_1^2 \dfrac{x^2}{\left(x^3+1\right)^2} dx$ (5 marks)

d) Differentiate the function and find the slope of the tangent line at the given value of the independent variable:

$y = x + \dfrac{9}{x}, \quad x = -3.$ (5 marks)

QUESTION FIVE (20 marks)

a) The population $P(t)$ of a bacterial colony t hours after observation begins is found to be changing at the rate:

$\dfrac{dP}{dt} = 200e^{0.1t} + 150e^{-0.03t}$

If the population was $200,000$ bacteria when the observations began, what will the population be 12 hours later? (5 marks)

b) Find the area enclosed between the two curves $y = 4 - x^2$ and $y = x^2 - 2x$ (7 marks)

c) An efficiency study of the morning shift at a certain factory indicates that an average worker who arrives on the job at $8.00 A.M.$ will have produced

$Q(t) = -t^3 + 6t^2 + 24t$

units t hours later:

i. Compute the worker's rate of production at $11.00 A.M$? (4marks)

ii. At what rate is the worker's rate of production changing with respect to time at $11.00 A.M$? (4 marks)