# JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

## UNIVERSITY EXAMINATION 2012/2013

## 1ST YEAR 2ND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION ARTS WITH IT

## (SCHOOL BASED)

**COURSE CODE: SMA 102**

**TITLE: CALCULUS I**

**DATE: 1/5/2013** **TIME: 3.30-5.30PM**

**DURATION: 2 HOURS**

**<u>INSTRUCTIONS</u>**

1. This paper contains FIVE (5) questions
2. Answer question 1 (Compulsory) and **ANY** other 2 Questions
3. Write all answers in the booklet provided

**QUESTIONS 1: [30 MARKS](COMPULSORY)**

(a) Evaluate the limits of the following:

(i) $\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3}$ (3 marks)

(ii) $\lim_{x \to 0} \frac{(x+1)^{1/2} - 1}{x}$ (3 marks)

(b) (i) Define the term continuity of a function $f(x)$ at the point $x = a$ (3 marks)

(ii)Discuss continuity of the function $f(x)$ at the given intervals when (4 marks)

$$f(x) = \begin{cases} 5 - x, & -1 \le x \le 2 \\ x^2 - 1, & 2 < x \le 3. \end{cases}$$

(c ) Find the derivatives of the given functions below from first principles.

(i) $f(x) = x^2 + 2x$ (4 marks)

(ii) $y = Sinx$ (4 marks)

(d) Determine the equation of the tangent to the curve $y = x^3 - 2x^2 - 2x$ at the point $(3,0)$.

(3 marks)

(e) Differentiate the function $y = (3x - 2x^2 + x^3)^6$ (6marks)

**QUESTION 2: [20 MARKS]**

(a) Given the function defined as $xy + x - 2y = 1$, find its derivative. (5 marks)

(b) Differentiate the following functions with respect to $x$;

(i) $y = \cos x(1 - \sin x)$. (4 marks)

(ii) $y = \ln \frac{(2x^2 - x + 2)}{(x^2 - x)}$ (4 marks)

(iii) $y = e^{-(x^2+1)}$ (4 marks)

(c) Show that the derivative of $\sin^{-1} x$ is $\frac{1}{\sqrt{1-x^2}}$ (3 marks)

**<u>QUESTION 3 :[20 MARKS]</u>**

(a) Consider the curve $f(x) = 5 + 24x - 9x^2 - 2x^3$

(i)Find the $y-$intercept (3 marks)

(ii) Determine $f'(x)$ and $f''(x)$. (2 marks)

(iii)Find the stationary points of the curve, hence distinguish between them.(5 marks)

(b) Given the equation $x^3 + 2xy - y^2 + x = 2$,

Find

(i) the slope of the curve at (-4, 1) and (3 marks)

(ii) the equation of the normal line to the curve at $(-4,1)$. (2 marks)

(c) A curve is defined parametrically by $x = 2\cos$¯ and $y = 2\sin$¯ ,find the slope of the curve when ¯ $= \frac{\text{®}}{3}$. (5 marks)

**<u>QUESTION 4: [20 MARKS)</u>**

(a) (i) Evaluate $\int \frac{x^2 + x + 1}{\sqrt{x}}\, dx$ (4 marks)

(ii)Evaluate $\int Sin \frac{1}{2} x \; dx.$ ( 3 marks)

(b) (i) Show that

$\frac{d}{dx}\left(\frac{1}{2}x - \frac{1}{4} Sin 2x\right) = Sin^2 x$. (3 marks) (ii)Differentiate the function $\frac{x}{\sqrt{(1+x^2)}}$ with respect to $x$ (4 marks)

(c ) A container in the shape of a right circular cone of height $10cm$ and base radius of $1cm$ is used in catching the drop from a tap leaking at the rate of $0.1cm^3/s$. Find the rate at which the surface area of the water is increasing when the water is halfway the cone.

(6 marks)

**QUESTION 5:[20 MARKS]**

(a) (i) Given that $\frac{dA}{dx} = \frac{(3x+1)(x^2-1)}{x^5}$; find A in terms of $x$. What is the value of A when $x = 2$, if $A = 0$ when $x = 1$. (5 marks)

(ii) A curve which passes through the point $(2,0)$ has a gradient function $\frac{3x^2-1}{x^2}$.. Find its equation. (3 marks)

(b) A two percent error is made in measuring the radius of a sphere. Find the percentage error in the surface area. (4 marks)

(c) A particle moves along a straight line $OA$ with a velocity of $(6-2t)m/s$. When $t = 1$, the particle is at O.

(i) Find an expression for its distance from 0 in terms of $t$. ( 3 marks)

(ii) Find the time at which it is momentarily at rest and hence calculate the actual distance through which it moves during the same time interval.

(5 marks)

**END**