# JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

## UNIVERSITY EXAMINATION 2012/2013

## 1ST YEAR 1ST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION ARTS WITH IT

## (SCHOOL BASED KOSELE LEARNING CENTRE)

**COURSE CODE: SMA 103**

**TITLE: LINEAR ALGEBRA I**

**DATE: 3/5/2013** **TIME: 3.30-5.30PM**

**DURATION: 2 HOURS**

**INSTRUCTIONS**

1. This paper contains FIVE (5) questions
2. Answer question 1 (Compulsory) and **ANY** other 2 Questions
3. Write all answers in the booklet provided

**QUESTION 1:(30 MARKS)[COMPULSORY]**

a) Determine the values of $x$ and $y$ that will make the vector $u$ and $v$ equal if $u = (x+2, y+4, -5)$ and $v = (4, 2, -5)$. (3marks)

b) Given that $u = (7,1,-3,6)$ and $v = (3,5,2-1)$ Find $\| u\text{-}v \|$. (3marks)

c) Find the cross product of the vectors (3marks)

$A = (1,-2,2)$ and $B = (0,1-3)$.

d) Find the distance of the point $A(25,5,7)$ from the plane $2x + 4y + 3z = 3$.

(3marks)

e) Given the systems of equations below

$x - 3z = -3,$

$2x + ky - z = -2,$

$x + 2y + kz = 1.$

Find the value of $k$ such that the equations have no solution. (4marks)

f) Show that $u+v,\ u-v, u-2v+w$ are independent given that $u, v, w$ are independent vectors. (3 marks)

g) Let $F : R^2 \rightarrow R^2$ be defined by $F(x, y) = (2x - y, x)$.

Determine whether or not F is linear. (4marks)

h) Write the vectors $v = (3,5)$ as a linear combination of $e_1 = (1,3)$ and $e_2 = (0,2)$

(3marks)

i) Let $u, w$ be the subspaces of $R^3$ defined by

$u = \{(a,b,c) : a = b = c\ \}$: $a = b = c$ and

$w = \{(0,b,c) : b, c \in R\ \}$: being a set of all vectors. Show that $R^3 = u + w.$

(4marks)

**<u>QUESTION 2:(20 MARKS)</u>**

a) Find the angle between the vectors $u = (2,4,5)$ and $v = (-1,3,2)$. (3marks)

b) Let u and v be a vector in an inner product space V, prove that

$\|u+v\| \leq \|u\| + \|v\|$. (6marks)

c) Find the equation of the plane through the point p (1, 2, 3) and is

perpendicular to the vector $4i + 5j + 6k$. (4marks)

d) Let $T : R^2 \to R^3$: be the linear mapping to which ;

$T(1,2) = (3,-1,5) and T(0,1) = (2,1,-1)$ (5marks)

(i) Find $T(a,b)$.

(ii) Evaluate $T(2,3)$ (2marks)

**QUESTION 3: (20MARKS)**

(a) Let V be the space of all 2x2 matrices over R. Let U and W be subspaces spanned by (A,B,C) and (D,E,F) respectively, where

$$A=\begin{pmatrix}4 & 1\\ -3 & 1\end{pmatrix}, B=\begin{pmatrix}2 & 2\\ -4 & -1\end{pmatrix}, C=\begin{pmatrix}2 & 5\\ -9 & -2\end{pmatrix}, D=\begin{pmatrix}3 & -2\\ 6 & 1\end{pmatrix}, E=\begin{pmatrix}-5 & -6\\ 8 & 2\end{pmatrix}, F=\begin{pmatrix}-6 & -5\\ 3 & 1\end{pmatrix}$$

i) Find dim $(u+w)$. (8marks)

ii) Find dim $(u \cap w)$. (2marks)

b) Let V be the vector space of polynominials of degree ≤ 3.Find, if possible non-zero scalars a, b and c such that $au+bv+cw=0,$ where:

$u=t^3-5t^2-2t+3$

$v=t^3-4t^2-3t+4$ and

$w=2t^3-7t^2-7t+9.$

Deduce whether $u,v$ and $w$ are linearly dependent or independent. (10marks)

**QUESTION 4 :( 20MARKS)**

a) i) Define the term basis as used in linear algebra. (2marks)

ii)Find the rank of the matrix (4marks)

$$A=\begin{pmatrix}1 & 3 & -2 & 5 & 4\\ 1 & 4 & 1 & 3 & 5\\ 1 & 4 & 2 & 4 & 3\\ 2 & 7 & -3 & 6 & 13\end{pmatrix}$$

b)Find the dimension and basis for the solution space of the homogenous systems of equations given below;

$x + 2y - 4z + 3r - s = 0,$

$x + 2y - 2z + 2r + s = 0,$

$2x + 4y - 2z + 3r + 4s = 0,$ (10marks)

c)Determine $k$ so that the vectors $u = (2, 3k, -4, 1, 5)$ and $v = (6, -1, 3, 7, 2k)$ are orthogonal (4marks)

## QUESTION 5: (20MARKS)

a)Define a linear Transformation $F$ from a linear space $v$ into a linear space $u$ (2marks)

b)Given that the operator $T$ on $R^3$ is defined by

$T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$. Find a Formula for the inverse operator $T^{-1}$. (6marks)

c)Let $F : R^4 \to R^3$ be the linear matrix

$F(x, y, s, t) = (x - y + s + t, x + 2s - t, x + y + 3s - 3t).$

i)Find the dimension of image of F. (3marks)

ii)Find a basis of the range of F. (4marks)

ii)Determine the kernel of F. (5marks)