COURSE TITLE: **LINEAR ALGEBRA II**

COURSE CODE: **SMA 201**; TIME: 2 HOURS

KOSELE LC; DECEMBER 2012

Answer question ONE and any other TWO questions.

QUESTION 1

(a) Determine whether A_1, A_2, and A_3 are linearly dependent or independent if; $A_1 = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & -1 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1 & 1 & 4 \\ 2 & 3 & 0 \end{bmatrix}$

and $A_3 = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix}$. **[5mks]**

(b) Show that v=$\{1\}$ is not a vector space. **[2mks]**

(c) Find a linear transformation from $\mathbb{R}^2$ into the plane

$w = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : 2x - y + 3z = 0 \right\}$. Hence find $T = \begin{pmatrix} 5 \\ -7 \end{pmatrix}$ given that

$w_1 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and $w_2 = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$. **[8mks]**

(d) In $\mathbb{R}^3$ show that $\begin{pmatrix} -7 \\ 7 \\ 7 \end{pmatrix}$ is a linear combination of $\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$ and

$\begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix}$. **[6mks]**

(e) Let $T : V \to W$ be a linear transformation.

Prove that $T(u - v) = Tu - Tv$

for all vectors $u, v \in V$. **[3mks]**

(f) If $\pi = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : 2x - y + 3z = 0 \right\}$ prove that the vectors in π have the form $\begin{pmatrix} x \\ 2x + 3z \\ z \end{pmatrix}$ hence a find the basis for the set of vectors lying on the plane. **[6mks]**

QUESTION 2

(a) Determine whether the three vectors in $\mathbb{R}^3$, $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 7 \end{pmatrix}$ are linearly dependent or independent. **[10mks]**

(b) Find a basis for the solution space of the homogenous systems given below

$$\begin{aligned} x + 2y - z &= 0 \\ 2x - y + 3z &= 0 \end{aligned}$$

Hence find the dimension of the solution space. **[10mks]**

QUESTION 3

(a) Determine whether the polynomials

$x - 2x^2; x^2 - 4x$ and $-7x + 8x^2$ are linearly dependent or independent, hence solve the homogenous systems. **[10mks]**

(b) If $v = (x, y, z) \in H$ and $v_1 = (2, -1, 4)$; $v_2 = (4, 1, 6)$ prove that $H = span\{v_1, v_2\} = \{v : v = a_1(2, -1, 4) + a_2(4, 1, 6)\}$. **[10mks]**

QUESTION 4

(a) Consider the set of vectors $w = \{(2,2,4); (0,4,10); (3,1,1)\}$ of $\mathbb{R}^3$, determine whether or not w is a linear independent set of vectors. **[10mks]**

(b) Let T be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ and suppose that $T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}; T\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}-1\\4\end{pmatrix}$

and $T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}5\\-3\end{pmatrix}$ compute $T\begin{pmatrix}3\\-4\\5\end{pmatrix}$. **[6mks]**

(c) Show that in M_{23}; $\begin{pmatrix}-3 & 2 & 8\\-1 & 9 & 3\end{pmatrix}$ is a linear combination of $\begin{pmatrix}-1 & 0 & 4\\1 & 1 & 5\end{pmatrix}$

and $\begin{pmatrix}0 & 1 & -2\\-2 & 3 & -6\end{pmatrix}$. **[10mks]**

QUESTION 5

(a) Determine the values of x and y that will make $\mathbf{u}$ and $\mathbf{v}$ equal if $\mathbf{u} = (x+1, 2, y-4, 5)$ and $\mathbf{v} = (4, 2, 0, -5)$ **[2mks]**

(b) The matrix M of a linear transformation T from $\mathbb{R}^3 \to \mathbb{R}^2$ is defined

by, $\begin{pmatrix}1 & -1 & 3\\2 & 0 & 4\\-1 & -3 & 1\end{pmatrix}$

Determine the Kernel of T. **[12mks]**

(c) Let M be $m \times n$ matrix and consider the mapping $T = \mathbb{R}^n \to \mathbb{R}^m$ defined by $T(x) = M(x)$ for every n-vectors x. Show that T is a linear transformation. **[6mks]**