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RESEARCH ARTICLE

BINOMIAL MIXTURES BASED ON BETA I DISTRIBUTION AND ITS GENERALIZATIONS WITH APPLICATION TO A TWO STAGE GROUP SCREENING DESIGN

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ABSTRACT

When mixed, two distributions can form another distribution. Binomial distribution and beta distributions combine to form a binomial mixture with the latter being a prior distribution which is a continuous distribution. Skellam pioneered this study when he mixed binomial distribution with its parameter p taking beta distribution. The paper focused on construction of binomial mixtures, their properties, special cases and the application of the mixtures in a two stage group screening design. The two methods of constructing mixtures were proved to yield identical results. The binomial mixtures obtained were expressed in recursive forms and their corresponding differential equations obtained. The moments obtained indicated that binomial mixtures are probability density functions. Applied to two stage group screening design, binomial mixtures have demonstrated reduced number of tests which are more cost effective than individual testing.

Key words: Binomial mixtures, recursive forms, group screening, identity, moments.

1.0 Introduction.

Let $f(x)$ be a function of a random variable X. If $\int_{-\infty}^{\infty} f(x)dx = 1$, then $f(x)$ is a probability density function of a continuous random variable X.

And if $\sum_{-\infty}^{\infty} f(x) = 1$ then $f(x)$ is a probability mass function of a discrete random variable X.

There are many methods of constructing $f(x)$. One of the methods is based on construction of mixtures. Mixed distributions can be generated by randomizing a parameter(s) in the parent distribution.Feller [4] proposed that if $f(i)$ and $g(i)$ are two probability distributions and if $\alpha > 0, \beta > 0$ and $\alpha + \beta = 1$, then$(\alpha fi + \beta gi)$ is also called a distribution. He called distributions generated in this manner, mixtures. Skellam [13] came up with an expression for the beta-binomial distribution. He studied its properties and suggested method of determining maximum likelihood estimates of the parameters of the beta mixing distribution. Ishi and Hayawaka [7] also derived the beta-binomial distribution and examined its properties and extensions.Binomial distribution has parameters n and p either and/or both of which may be randomized to give a binomial mixture. This paper discusses a case in which parameter p has a continuous mixing distribution with probability density $g(p)$ so that $f(x) = \int \binom{n}{x} p^x (1-p)^{n-x} g(p)dp$., where $f(x)$ is the binomial mixture. The objective of this paper is to determine beta-binomial mixture using methods of integration and moments. It will also express beta-binomial mixtures obtained in a recursive form. The moments of the beta-binomial mixtures will be determined. And the binomial mixtures will be applied in a two stage group screening design.

2.0 Method

2.1 Formulation of the problem set.

The expression for the continuous mixture is

$$f(x) = \int f(x \mid \theta) g(\theta) d\theta$$

where $f(x)$ is the mixed distribution, $f(x \mid \theta)$ is the conditional distribution and $g(\theta)$ is a continuous mixing distribution.

The continuous beta- binomial mixture is given as

$$f(x) = \int_0^1 \binom{n}{x} p^x (1-p)^{n-x} g(p)dp \qquad \text{....... [1]}$$

Two methods of construction are applicable. These include;

i) Explicit form/ Direct Integration which is

$$f(x) = \int_0^1 \binom{n}{x} p^x (1-p)^{n-x} g(p)dp$$

$$= \int_0^1 \binom{n}{x} p^x (1-p)^{n-x} \frac{p^{a-1}(1-p)^{b-1}}{B(a,b)} dp$$

$$= \binom{n}{x} \frac{1}{B(a,b)} \int_0^1 p^{x+a-1} (1-p)^{n-x+b-1} dp$$

$$f(x) = \frac{B(x+a, n-x+b)}{B(a,b)} \qquad \text{.......[2]}$$

ii) Expectation form which is

$$f(x) = \int_0^1 \binom{n}{x} p^x (1-p)^{n-x} g(p)dp$$

$$= \int_0^1 \binom{n}{x} p^x \sum_{k=0}^{n-x} \binom{n-x}{k} (-p)^k g(p)dp$$

$$f(x) = \binom{n}{x} \sum_{j=x}^{n} (-1)^{j-k} \binom{n-x}{j-x} E(P^j) . \qquad ....[3]$$

This method is due to Sivaganesan and Berger[9].

Beta-Binomial distribution obtained above can be expressed in recursive form. This can be achieved using ratio method and integration by parts techniques.

Ratio method involves expressing $f(x) = \dfrac{B(x+a, n-x+b)}{B(a,b)}$ as $\dfrac{f(x+1)}{f(x)}$ while in integration by parts technique, $f(x) = \dfrac{B(x+a, n-x+b)}{B(a,b)}$ is expressed using the model $\int udv = uv - \int vdu$.

Beta-Binomial distribution can as well be fitted into other models such as Panjer-Willmot[10] and Hesselager's [7] models.Panjer-Willmot recursive model is given as

$$f(x)\sum_{t=0}^{k} \alpha_t x^{(t)} = f(x-1)\sum_{t=0}^{k} \beta_t (x-1)^{(t)}$$

where $x^{(t)} = x(x-1)(x-2)...(x-t+1)$

and $(x-1)^{(t)} = (x-1)(x-2)...(x-t).$

For k=2, we have

$$[\alpha_0 + \alpha_1 x + \alpha_2 x(x-1)] f(x) = [\beta_0 + \beta_1 (x-1) + \beta_2 (x-1)(x-2)] f(x-1). \qquad ....[4]$$

The differential equation in probability generating function is

$$\sum_{t=1}^{k} \alpha_t s^t G^{(t)}(s) + s\sum_{t=1}^{k} \beta_t s^t G^{(t)}(s) + (\alpha_0 - \beta_s) G(s) = \alpha_0 p_0 \qquad ....[5]$$

And Hesselager's recursive model is given as

$$f(x)\sum_{i=0}^{k} b_i x^i = f(x-1)\sum_{i=0}^{k} a_i x^i$$

For k=1 we have

$$[b_0 + b_1 x] f(x) = [c_0 + c_1 (x-1)] f(x-1), \quad x = 1,2,3...$$

The corresponding differential equation is

$$s[b_1 - c_1 s]G'(s) + (b_0 + c_0 s)G(s) = b_0 p_0 \qquad ....[6]$$

When k=2 we have

$$f(x) = \sum_{i=0}^{2} b_i x^i = f(x-1)\sum_{i=0}^{2} c_i (x-1)^i$$

The corresponding differential equation becomes

$$s[b_1 - c_1 s]G''(s) + s\{(b_1 + b_2) - (c_1 + c_2)s\}G'(s) + (b_0 - c_0 s)G(s) = b_0 p_0 . \qquad ....[7]$$

Using the Probability generating function technique moments of Beta-Binomial distribution can be obtained.

Let

$$G(1+s) = \sum_{x=0}^{n} f(x)(1+s)^x$$

$$= \sum_{x=0}^{n} \int_0^1 \binom{n}{x} p^x (1-p)^{n-x} g(p)(1+s)^x \, dp$$

$$= \int_0^1 [1-p+p(1+s)]^n g(p)dp$$

$$G(1+s) = \int_0^1 (1+ps)^n g(p)dp$$

$$G'(1+s) = \int_0^1 np(1+ps)^{n-1} g(p)dp$$

and

$$G^{(r)}(1+s) = \int_0^1 p^r n^{(r)} (1+ps)^{n-r} g(p)dp$$

Therefore

$$E[X(X-1)(X-2)...(X-r+1)] = G^{(r)}(1+s)$$

$$= \int_0^1 p^r n^{(r)} g(p)dp$$

$$= n^{(r)} E[P^r], \qquad ....[8]$$

where $n^{(r)} = n(n-1)(n-2)...(n-r+1)$.

Therefore

$$E(X) = nE(P) \qquad ....[9]$$

$$Var(X) = Var(P) + nE(P) - nE(P^2) \qquad ....[10]$$

Note that the above problem set is to be used:

- To determine binomial mixtures by direct method and by method of moments andthen to prove the identity obtained by equating the two equivalent results.
- To express binomial mixtures in recursive form.
- To obtain moments of the binomial mixtures.

2.2 Formulation of the second problem set.

A two stage group screening design due to Dorfman[4] can be formulated as follows:

Let $N =$ the total number of individuals to be tested divided into *g* groups, each of size *k*.

Therefore $N = kg$

A group is positive if at least one individual is positive, each with probability p , varying according to a certain distribution.

Let S be the number of positive individuals in a group of size k. Then,

$$\Pr ob(S \geq s) = \binom{k}{s} p^s (1-p)^{k-s}, \quad s = 0,1,2...k$$

The probability of a group being positive is

$$\Pr ob(S \geq 1) = \sum_{s=1}^{k} \binom{k}{s} p^s (1-p)^{k-s}$$

$$= 1 - \Pr ob(S = 0)$$

$$= 1 - (1-p)^k \qquad ....[11]$$

Which can be labeled as p^*

If r is the number of positive groups out of g groups, then the distribution of r given p is a binomial distribution with parameter p^* i.e,

$$f(r \mid p) = \binom{g}{r} p^{*r}(1-p^{*r})^{g-r}, \quad r = 0,1,2\ldots g. \qquad \text{.......[12]}$$

Therefore

$$f(r) = \int_0^1 \binom{g}{r} p^{*r}(1-p^*)^{g-r} g(p)dp.$$

Therefore

$$E(R) = EE(R \mid P)$$
$$= E(gP^*)$$
$$\vdots$$
$$= g\sum_{j=1}^{k}(-1)^{j+1}\binom{k}{j}E(P^j)$$

For the two stage design,

Stage	Number of Tests
I	g groups
II	kr individuals
TOTAL(T)	$g + kr$

Therefore, $T = g + kr$.

And

$$E(T) = g + kE(R)$$
$$= \tfrac{N}{k} + N\sum_{j=1}^{k}(-1)^{j+1}\binom{k}{j}E(P^j)$$
$$= N\left\{\tfrac{1}{k} + \sum_{j=1}^{k}(-1)^{j+1}\binom{k}{j}E(P^j)\right\} \qquad \text{.......[13]}$$

Problem is to determine $E(T)$

3.0 Results

3.1 Beta-Binomial Distribution

The history of beta-Binomial distribution goes back to Skellam [13] when he obtained the binomial mixture treating the parameter *p,* the probability of success as random variable of beta distribution of the first kind.

Beta I (classical Beta) Mixing distribution is

$$g(p) = \frac{p^{a-1}(1-p)^{b-1}}{B(a,b)}, \quad 0 < p < 1; \quad a,b > 0 \qquad \text{.......[14]}$$

The j[th] moment of beta mixing distribution is

$$E(P^j) = \frac{1}{B(a,b)}\int_0^1 p^{j+a-1}(1-p)^{b-1}$$

$$E(P^j) = \frac{B(j+a,b)}{B(a,b)}. \qquad \text{.......[15]}$$

Implying the expected value of beta mixing distribution is

$$E(P)=\frac{B(1+a,b)}{B(a,b)}$$

$$=\frac{a}{a+b} \qquad \text{.......[16]}$$

$$E(P^2)=\frac{a(a+1)}{(a+b)(a+b+1)}$$

And

$$Var(P)=E(P^2)-[E(P)]^2$$

$$=\frac{a(a+1)}{(a+b)(a+b+1)}-\left[\frac{a}{a+b}\right]^2$$

Therefore

$$Var(P)=\frac{ab}{(a+b)^2(a+b+1)} \qquad \text{.......[17]}$$

The probability mass function of the binomial mixture becomes, by direct integration,

$$f(x)=\binom{n}{x}\frac{B(a+x,n-x+b)}{B(a,b)}, \quad x=0,1,2\ldots n \qquad \text{.......[18]}$$

And by method of moments,

$$f(x)=\binom{n}{x}\sum_{j=x}^{n}(-1)^{j-x}\binom{n-x}{j-x}\frac{B(j+a,b)}{B(a,b)}\ . \qquad \text{.......[19]}$$

3.2 Identity

The two methods of binomial construction, that is, method of moments and explicit produce two results which can be proved to be identical.

$$\sum_{j=x}^{n}(-1)^{j-x}\binom{n-x}{j-x}B(j+a,b)=B(a+x,n-x+b) \text{ [20]}$$

Taking LHS, we have

$$\sum_{j=x}^{n}(-1)^{j-x}\binom{n-x}{j-x}B(j+a,b)$$

$$=\sum_{k=0}^{n-x}(-1)^k p^k\int_0^1 p^{x+a-1}(1-p)^{b-1}dp$$

$$=\int_0^1 p^{x+a-1}(1-p)^{n-x+b-1}dp$$

$=B(x+a,n-x+b)$, which is the RHS of equation ....[20]

In a recursive form the binomial mixture can be expressed as

$$f(x+1)=\frac{(n-x)(a+x)}{(x+1)(n-x+b-1)}f(x), \quad x=0,1,2,\ldots n$$

$$\left[(n+b)(x+1)-(x+1)^2\right]f(x+1)=\left[na+(n-a)x-x^2\right]f(x) \qquad x=0,1,2,..n \qquad \text{.......[21]}$$

The corresponding differential equation in probability generating function is

$$s(1-s)G''(s)-[(n+b-1)-(n-a-1)s]G'(s)+naG(s)=0 . \quad ....[22]$$

Expressed in hypergeometric form beta-binomial distribution becomes

$$f(x)=\binom{n}{x}\frac{B(a+x,n+b-x)}{B(a,b)}$$

$$=\frac{n!}{x!(n-x)!}\frac{\Gamma(x+a)\Gamma(n-x+b)\Gamma(a+b)}{\Gamma(n+a+b)\Gamma a\Gamma b}$$

$$\vdots$$

$$=\frac{(x+a-1)!(n-x+b-1)!n!(a+b-1)!}{x!(a-1)!(n-x)!(b-1)!(n+a+b-1)!}$$

Therefore

$$f(x)=\frac{\binom{-a}{x}\binom{-b}{n-x}}{\binom{-(a+b)}{n}}, \quad x=0,1,2,...n , \quad ....[23]$$

which justifies the name of negative hypergeometric distribution given to beta-binomial distribution by Shelton(1950).

3.3 Moments

The mean of beta-binomial distribution is

$$E(X)=nE(P)$$

$$E(X)=\frac{na}{a+b} \quad ....[24]$$

And the variance becomes

$$Var(X)=\frac{nab(n+a+b)}{(a+b)^2(a+b+1)} . \quad ....[25]$$

3.4Group screening

For the two stage group screening the probability distribution of r defective groups is

$$f(r)=\int_0^1\binom{g}{r}p^{*r}(1-p^*)^{g-r}\,\frac{p^{a-1}(1-p)^{b-1}}{B(a,b)}dp$$

$$=\binom{g}{r}\sum_{l=0}^{r}(-1)^l\binom{r}{l}\frac{B(a,kl+kg-kr+b)}{B(a,b)} \quad ....[26]$$

The expected number of defective groups are

$$E(R)=g\sum_{j=1}^{k}(-1)^{j+1}\binom{k}{j}\frac{B(j+a,b)}{B(a,b)}$$

The expected total number of tests done are

$$E(T)=N\left\{\frac{1}{k}+\sum_{j=1}^{k}(-1)^{j+1}\binom{k}{j}\frac{B(j+a,b)}{B(a,b)}\right\}. \quad ....[27]$$

4.0 Special cases of Beta-Binomial distribution

4.1 *Uniform-Binomial distribution:*

Binomial-Uniform distribution is obtained when $a=b=1$.

The uniform mixing distribution is

$$g(p) = 1, 0 < p < 1 \quad ....[28]$$

And the j[th] moment becomes

$$E(P^j) = \frac{1}{j+1} \quad [29]$$

Therefore

$$E(P) = \frac{1}{2} \quad and \quad E(P^2) = \frac{1}{3}$$

$$Var(P) = \frac{1}{12} \quad ....[30]$$

By direct integration the probability mass function of binomial-uniform distribution is

$$f(x) = \frac{1}{n+1}, \quad x = 0,1,2,...n, \quad ....[31]$$

which is discrete uniform distribution.

And by method of moments, we have

$$f(x) = \binom{n}{x}\sum_{j=x}^{n}(-1)^{j-x}\binom{n-x}{j-x}\frac{1}{j+1}.$$

Their identity is

$$\binom{n}{x}\sum_{j=x}^{n}(-1)^{j-x}\binom{n-x}{j-x}\frac{1}{j+1} = \frac{1}{n+1} \quad ....[32]$$

Expressed in recursive form binomial-uniform distribution becomes

$$f(x) = f(x-1), \quad x = 1,2,...n \quad ....[33]$$

The corresponding differential equation:

$$s(1-s)G''(s) - [n-(n-2)s]G'(s) + nG(s) = 0 \quad ....[34]$$

In hypergeometric form we obtain

$$f(x) = \frac{\binom{-1}{x}\binom{-1}{n-x}}{\binom{-2}{n}}, \quad x = 0,1,2,...n. \quad ....[35]$$

The mean of binomial-uniform distribution is

$$E(X) = \frac{n}{2} \quad ....[36]$$

And the variance becomes

$$Var(X) = \frac{n(n+2)}{12} \quad ....[37]$$

For two stage group screening, we have

$$f(r) = \sum_{l=0}^{r}(-1)^l\binom{r}{l}\binom{g}{r}B(1, kl+kg-kr+1)$$

$$= \sum_{l=0}^{r}(-1)^l\binom{r}{l}\binom{g}{r}\frac{1}{kl+kg-kr+1}$$

Thus the expected number of defective groups are

$$E(R) = g\sum_{j=1}^{k}(-1)^{j+1}\binom{k}{j}B(j+1,1)$$

$$= g\sum_{j=1}^{k}(-1)^{j+1}\binom{k}{j}\frac{1}{j+1}$$

And the total number of tests done are

$$E(T) = N\left\{\frac{1}{k} + k\sum_{j=1}^{k}(-1)^{j+1}\binom{k}{j}B(j+1,1)\right\}$$

$$= N\left\{\frac{1}{k} + \sum_{j=1}^{k}(-1)^{j+1}\binom{k}{j}\frac{1}{j+1}\right\}. \quad ....[38]$$

4.2*Binomial-Power distribution:*

Binomial power distribution is obtained when $b = 1$.

The probability mass function for power mixing distribution is

$$g(p) = ap^{a-1}, \quad 0 < p < 1;\ a > 0 \quad ....[39]$$

The j[th] moment for the mixing distribution is

$$E(P^j) = \frac{1}{j+a}. \quad ....[40]$$

$$E(P) = \frac{a}{a+1} \quad ....[41]$$

$$E(P^2) = \frac{a}{(a+1)(a+2)}$$

And the variance is

$$Var(P) = \frac{a}{(a+1)^2(a+2)}. \quad ....[42]$$

The binomial-power distribution becomes

$$f(x) = a\binom{n}{x}B(a+x, n-x+1)$$

Also

$$f(x) = a\binom{n}{x}\sum_{j=x}^{n}(-1)^{j-x}\binom{n-x}{j-x}\frac{1}{a+j}$$

Identity:

$$B(a+x, n-x+1) = \sum_{j=x}^{n}(-1)^{j-x}\binom{n-x}{j-x}\frac{1}{a+j} \quad ....[43]$$

In recursive form, we have

$$f(x+1) = \frac{x+a}{x+1}f(x), \quad x = 0,1,2,...n$$

$$[x+1]f(x+1) = [x+a]f(x). \quad ....[44]$$

Andthe corresponding differential equation becomes

$$s(1-s)G''(s) - [n-(n-a-1)s]G'(s) + naG(s) = 0 \quad ....[45]$$

In hypergeometric form we have

$$f(x)=\frac{\binom{-a}{x}\binom{-1}{n-x}}{\binom{-(a+1)}{n}}, \quad x=0,1,2,\ldots n \qquad \text{.......[46]}$$

Moments:

The expectation of binomial-power distribution is

$$E(X)=\frac{na}{a+1} \qquad \text{.......[47]}$$

$$Var(X)=\frac{na(n+a+1)}{(a+1)^2(a+b+1)} \qquad \text{.......[48]}$$

For the two stage group screening we have

$$f(r)=a\binom{g}{r}\sum_{l=0}^{r}(-1)^l\binom{r}{l}B(a,kl+kg-kr+1)$$

And the total expected number of tests are

$$E(T)=N\left\{\frac{1}{k}+\sum_{j=1}^{k}(-1)^{j+1}\binom{k}{j}\frac{a}{j+a}\right\} \qquad \text{.......[49]}$$

4.3 ***Binomial-Arc- Sine Distribution:***

Binomial-Arc Sine distribution is obtained when $a=b=\frac{1}{2}$.

The Arc-Sine mixing distribution is given as

$$g(p)=\frac{p^{-\frac{1}{2}}(1-p)^{-\frac{1}{2}}}{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2})}$$

$$=\frac{1}{\pi}.\frac{1}{\sqrt{p(1-p)}} \qquad \text{.......[50]}$$

The j[th] moment of Arc-sine mixing distribution is

$$E(P^j)=\frac{\Gamma(j+\frac{1}{2})}{j!\Gamma(\frac{1}{2})}\ , \qquad \text{.......[51]}$$

implying the mean is

$$E(p)=\frac{\Gamma(1+\frac{1}{2})}{1\Gamma\frac{1}{2}}=\frac{\frac{1}{2}\Gamma\frac{1}{2}}{\Gamma\frac{1}{2}}=\frac{1}{2} \qquad \text{.......[52]}$$

$$E(P^2)=\frac{3}{8}.$$

The variance becomes

$$Var(P)=\frac{1}{8}. \qquad \text{....[53]}$$

The probability mass function of Binomial-Arc-Sine is, by direct method

$$f(x)=\binom{n}{x}\frac{B(x+\frac{1}{2},n-x+\frac{1}{2})}{B(\frac{1}{2},\frac{1}{2})}$$

Also by method of moments

$$f(x)=\binom{n}{x}\sum_{j=x}^{n}(-1)^{j-x}\binom{n-x}{j-x}\frac{B(j+\frac{1}{2},\frac{1}{2})}{B(\frac{1}{2},\frac{1}{2})}$$

The Identity of Binomial-Arc-Sine is

$$\sum_{j=x}^{n}(-1)^{j-x}\binom{n-x}{j-x}B(j+\tfrac{1}{2},\tfrac{1}{2})=B(x+\tfrac{1}{2},n-x+\tfrac{1}{2}) \qquad \text{....... [54]}$$

Expressed in recursive form we get

$$f(x+1)=\frac{(n-x)(x+\frac{1}{2})}{(x+1)(n-x+\frac{1}{2})}f(x), \quad x=0,1,2,...n \qquad \text{....... [55]}$$

with $f(n+1)=0$.

And the corresponding differential equation becomes

$$s(1-s)G''(s)-\left[(n+\tfrac{3}{2})-(n-\tfrac{3}{2})s\right]G'(s)+\frac{n}{2}G(s)=0 \qquad \text{....... [56]}$$

Hypergeometric form of Binomial-Arc-Sine is

$$f(x)=\frac{\binom{-\frac{1}{2}}{x}\binom{-\frac{1}{2}}{n-x}}{\binom{-1}{n}}, \quad x=0,1,2,...n \qquad \text{....... [57]}$$

Moments:

The mean of Binomial-Arc-Sine is

$$E(X)=\frac{n}{2} \qquad \text{....... [58]}$$

And the variance becomes

$$Var(X)=\frac{n(n+1)}{8}. \qquad \text{....... [59]}$$

For the two stage group screening design

$$f(r)=\binom{g}{r}\sum_{l=0}^{r}(-1)^{l}\binom{r}{l}\frac{B(\frac{1}{2},kl+kg-kr+\frac{1}{2})}{B(\frac{1}{2},\frac{1}{2})}$$

And the total number of tests are

$$E(T)=N\left\{\frac{1}{k}+\sum_{j=1}^{k}(-1)^{j+1}\binom{k}{j}\frac{B(j+\frac{1}{2},\frac{1}{2})}{B(\frac{1}{2},\frac{1}{2})}\right\} \qquad \text{....... [60]}$$

5.0 Discussion

When treated as random variable, parameter p from the binomial distribution takes beta prior distribution to form a continuous random variable. And the mixed distribution obtained becomes a random variable with the domain (0,1). Both methods of construction of the binomial mixtures, that is, explicit and expectation forms proved to produce results that are identical. The moments of the binomial mixtures are obtained using probability generating function technique. In a two stage group screening, it is noted that the number of tests done in the second stage are more than the number of tests done in the first stage since the defective groups are retested involving each member of the group. There is a strong relationship between binomial mixtures and negative hypergeometric distribution. This relation is applicable to special cases of beta-binomial distribution such as Uniform-Binomial distribution, Binomial-Power distribution and Binomial-Arc-Sine distributions.

6.0 Conclusion

Binomial mixtures demonstrate a good example of mixed distributions. It addresses the challenge of over-dispersion that is caused by the randomized parameter p from the binomial distribution. The mixed distribution obtained can be applied in group testing design, aprocess which is cost effective and more precise in estimating disease prevalence. The binomial mixtures can be extended to include generalized beta distributions as mixing distributions.

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