JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

2ND YEAR 1ST SEMESTER 2019/2020 ACADEMIC YEAR

MAIN

REGULAR

COURSE CODE: SPH 203

COURSE TITLE: Mathematical Methods For Physics I

EXAM VENUE: **STREAM: EDUCATION**

DATE: **EXAM SESSION:**

TIME: 2:00 HRS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION 1 (30 MARKS)

a) Distinguish between a sequence and a series. **(2 marks)**

b) Prove that the sequence $\{a_n\} = \frac{1}{n}$ is Cauchy. **(3 marks)**

c) Determine the limit of the sequence $\ln\left(\frac{4n+1}{3n-1}\right)$ **(2 marks)**

d) Find the radius of convergence for the power series defined by $\sum_{k=1}^{\infty} k!(x+10)^k$ **(3 marks)**

e) The partition function of a spin-$\frac{1}{2}$ paramagnetic system is given by the exponential function

$Z = e^{\left(\frac{g_s \mu_B B}{2k_B T}\right)} + e^{-\left(\frac{g_s \mu_B B}{2k_B T}\right)}$. Express the equation as a hyperbolic function. **(2 marks)**

f) Obtain the derivative of the function $y = \sin 5x + \ln x^3 - e^{4x} + \sum_{k=0}^{\infty} c_k x^k$ **(4 marks)**

g) By using the method of integration by parts, evaluate $\int_0^N \ln x dx$ to obtain the Stirling's approximation for a system of N particles. **(3 marks)**

h) State the antiderivative form of the fundamental theorem of Calculus. (**1 mark)**

i) A physical system traces a path defined by the following parametric equations during its motion. $x = 2t^3 + 1$ and $y = t^2 \cos t$. Determine the gradient function of the path. **(3 marks)**

j) Calculate the work done when an object is pushed by a force $\vec{F} = (10\hat{i} + 15\hat{j})N$ through a displacement $\vec{s} = (3\hat{i} + 4\hat{j})m$. **(2 marks)**

k) Define the following terms.

(i) Linear independence. **(1 mark)**

(ii) Basis. **(1 mark)**

(iii) Asymptotic analysis **(1 mark)**

l) The unitless equivalents of Pauli spin matrices are given by $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$;

$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ and $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ where $i = \sqrt{-1}$. Show that the commutation bracket

$\left[\sigma_x, \sigma_y\right] = 2i\sigma_z$ **(2 marks)**

SECTION B

Attempt any TWO questions in this section.

QUESTION 2 (20 MARKS)

a)Determine $\lim_{x\to\infty}\left(\frac{6x^2+5x+7}{4x^2+2x}\right)$ using de l'H$\hat{o}$pital's rule. **(4 marks)**

b) Derive the expression for the vector product $\vec{A}\times\vec{B}$ where $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$;

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ **(6 marks)**

c) Given $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{B} = 3\hat{i} + 2\hat{j} + \hat{k}$. Find:

(i) $\vec{A}X\vec{B}$ **(3 marks)**

(ii) $\vec{A}.\vec{B}$ **(2 marks)**

(iii) Angle between $\vec{A}$ and $\vec{B}$ **(2 marks)**

(iv) Unit vector perpendicular to both $\vec{A}$ and $\vec{B}$. **(3 marks)**

QUESTION 3 (20 MARKS)

a) Apply the concept of integration of rational functions to evaluate

$$\int \frac{x^2+2x+4}{(x+1)^3}dx$$ **(6 marks)**

b) A particle described by the wave function $u=\frac{1}{\sqrt{a}}\cos\left(\frac{\pi x}{2a}\right)$ moves in a one-dimensional potential defined by $V=V_0\cos\left(\frac{\pi x}{2a}\right)$. Calculate the first order energy correction given by

$$E_1=\int_{-a}^{a}uVudx$$ **(14 marks)**

QUESTION 4 (20 MARKS)

a) Expand $f(x)=\sin x$ in a Taylor series about $\frac{\pi}{3}$ **(6 marks)**

b) Show that the series $\sum_{k=1}^{\infty}ar^{k-1}$ converges to $\frac{a}{1-r}$ if $|r|<1$ and diverges if $|r|\geq 1$ **(6 marks)**

c) A ladder $20\ m$ leans against a vertical wall. The foot of the ladder is being drawn away from the wall at the rate of $2\ m/s$. How fast is the top of the ladder sliding down at the instant when the foot is $10\ m$ from the wall? **(8 marks)**

QUESTION 5 (20 MARKS)

a) Use the substitution method to evaluate $\int 3\cos 3t dt$ **(4 marks)**

b) The primitive translation vectors of the body centred cubic (bcc) crystal lattice are given by

$\vec{a} = \frac{1}{2}a(\hat{x} + \hat{y} - \hat{z})$; $\vec{a} = \frac{1}{2}a(-\hat{x} + \hat{y} - \hat{z})$; $\vec{a} = \frac{1}{2}a(\hat{x} - \hat{y} + \hat{z})$. Calculate the volume of the primitive cell using the formula $V_{bcc} = \left|\vec{a}.\vec{b} \times \vec{c}\right|$ **(8 marks)**

c) (i) Define a linear map. **(2 marks)**

(ii)Let a map T be defined by $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$. Show that T is linear. **(6 marks)**